AUTOMATIC CONTROL SYSTEMS FOR LONGITUDINAL MOTION OF SEMISUBMERGED SHIPS

Gilbert Finley Monell

Library
U.S. Naval Postgraduate School
Monterey, California 93940

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AUTOMATIC CONTROL SYSTEMS
FOR
LONGITUDINAL MOTION OF SEMISUBMERGED SHIPS

by

Gilbert Finley Monell, Jr.

Thesis Advisor: D. E. Kirk

September 1972

Approved for public release; distribution unlimited.

149341

Automatic Control Systems
for
Longitudinal Motion of Semisubmerged Ships

by

Gilbert Finley Monell, Jr.
Lieutenant, United States Navy
BSEE, Purdue University, 1966

Submitted in partial fulfillment of the
requirements for the degree of

ELECTRICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
September 1972

Thesis
M673
c. 1

L
U School
Monterey, California 93940

ABSTRACT

Optimal control theory is used to develop three automatic control systems for the longitudinal motion of a semisubmerged ship. A linearized mathematical model of the ship motion is used and the control problem is treated as a linear regulator. Simulations of the ship's longitudinal motions, utilizing the three control systems, are compared for various sea conditions. It is concluded that if the wave forces and moments are known or are estimated as functions of time, a suboptimal controller is the best controller. However, if a frequency domain approach is used to estimate the wave forces and moments, the complexity of a suboptimal controller approaches that of an optimal controller and the difference between the two is negligible. Preliminary work on three techniques for estimating the forces and moments is presented.

TABLE OF CONTENTS

I. INTRODUCTION ---------- 7

II. PROBLEM FORMULATION ---------- 10

A. NON-DIMENSIONALITY ---------- 10

B. SEMISUBMERGED SHIP FORM USED FOR THEORETICAL ANALYSIS ---------- 11

C. COORDINATE SYSTEM ---------- 11

1. X_s - Axis ---------- 14

2. Y_s - Axis ---------- 15

3. Z_s - Axis ---------- 15

4. Angles of Rotation ---------- 15

D. LONGITUDINAL RESPONSE PROBLEM ---------- 15

III. SIMULATION MODEL ---------- 17

A. LINEARIZED EQUATIONS OF MOTION ---------- 17

B. TRANSFER FUNCTIONS ---------- 20

C. COMPUTER MODEL ---------- 22

1. Analog Model ---------- 22

2. Digital Model ---------- 25

IV. CONTROL SYSTEMS FOR THE SEMISUBMERGED SHIP ----- 28

A. PERFORMANCE MEASURE ---------- 28

B. CONTROL SYSTEM FOR THE SHIP IN A CALM SEA -- 29

C. THE OPTIMAL CONTROL LAW FOR THE SHIP WHEN SUBJECTED TO WAVE FORCES ---------- 30

D. A SUBOPTIMAL CONTROL LAW FOR THE SHIP WHEN SUBJECTED TO WAVE FORCES ---------- 35

E. CONSTANT STATE FEEDBACK ---------- 38

V. SIMULATION PROCEDURE ---------- 46

A. WAVE MODEL ---------- 46

B. COMMAND SIGNAL ---------- 48

VI. RESULTS ---------- 53

A. CASES INVESTIGATED ---------- 53

1. Case I ---------- 56

2. Case II ---------- 56

3. Case III ---------- 56

4. Case IV ---------- 56

5. Case V ---------- 56

6. Case VI ---------- 56

B. THE EFFECT OF VARYING THE CONTROL WEIGHTING MATRIX (CASE I) ---------- 56

C. CONSTRAINED CONTROL SURFACE DEFLECTIONS (CASE II) ---------- 57

D. THE CONSTANT-GAIN CONTROLLER (CASE III) ------ 64

E. THE OPTIMAL CONTROLLER (CASE IV) ---------- 68

F. THE SUBOPTIMAL CONTROLLER (CASE V) ---------- 68

G. THE EFFECT OF INACCURATE ESTIMATION OF WAVE FORCES (CASE VI) ---------- 68

VII. PRELIMINARY WORK ON WAVE ESTIMATORS ---------- 81

A. AN INPUT-SIGNAL OBSERVER ---------- 81

B. FAST-FOURIER TRANSFORM ESTIMATION ---------- 84

C. STATE OBSERVER MODEL ---------- 85

VIII. CONCLUSIONS ---------- 89

A. THE CONTROLLER ---------- 89

B. RECOMMENDATIONS FOR FURTHER STUDY ---------- 90

1. Sea Conditions ---------- 90

2. Control Dynamics ---------- 9

3. Measurement Noise ---------- 90

4. Wave Estimator ---------- 90

5. Weighting Matrices ---------- 91

6. Wave Contouring ---------- 91

APPENDIX A ---------- 92

APPENDIX B ---------- 94

COMPUTER PROGRAM ---------- 95

LIST OF REFERENCES ----------110

INITIAL DISTRIBUTION LIST ----------111

FORM DD 1473 ----------112

ACKNOWLEDGEMENT

A significant debt of gratitude is owed to Dr. D. E. Kirk, Naval Postgraduate School, for the many hours, ideas and assistance he extended in the preparation of this thesis. A special thanks goes to Dr. H. A. Titus, Naval Postgraduate School, for suggesting the area of investigation, and to Dr. D. T. Higdon, Naval Undersea Research and Development Center, for providing the specific problem areas and data of the semisubmerged ship.

I. INTRODUCTION

In 1968 the need arose for a small, relatively inexpensive ship which could travel as fast as large Naval vessels and provide support for one of the Naval Undersea Research and Development Center (NUC) projects -- an unmanned undersea vehicle. The ensuing investigation conducted by NUC resulted in the semisubmerged ship concept shown in Fig. 1 [Ref. 1].

Referring to Fig. 1, the concept is described as follows:

a. The two streamlined hulls provide the primary buoyancy and have less drag and experience smaller wave-induced forces than conventional ship hulls.

b. A propeller is placed at the end of each hull where efficiency is increased by utilizing the boundary layer inflow.

c. Two vertical, surface-piercing bouyant struts are attached to each hull, providing hydrostatic stability in pitch and heave due to the waterplane areas and spacing. Hydrodynamic stability in yaw is obtained from the hydrofoil behavior of the struts.

d. The rudders are placed in the propeller slipstreams which provides additional rudder force and allows the rudders to induce forces on the aft struts.

e. A horizontal tail surface provides pitch damping as well as pitch stability at high speeds.

f. The superstructure, aside from providing the normal equipment and personnel spaces, provides the juncture for the

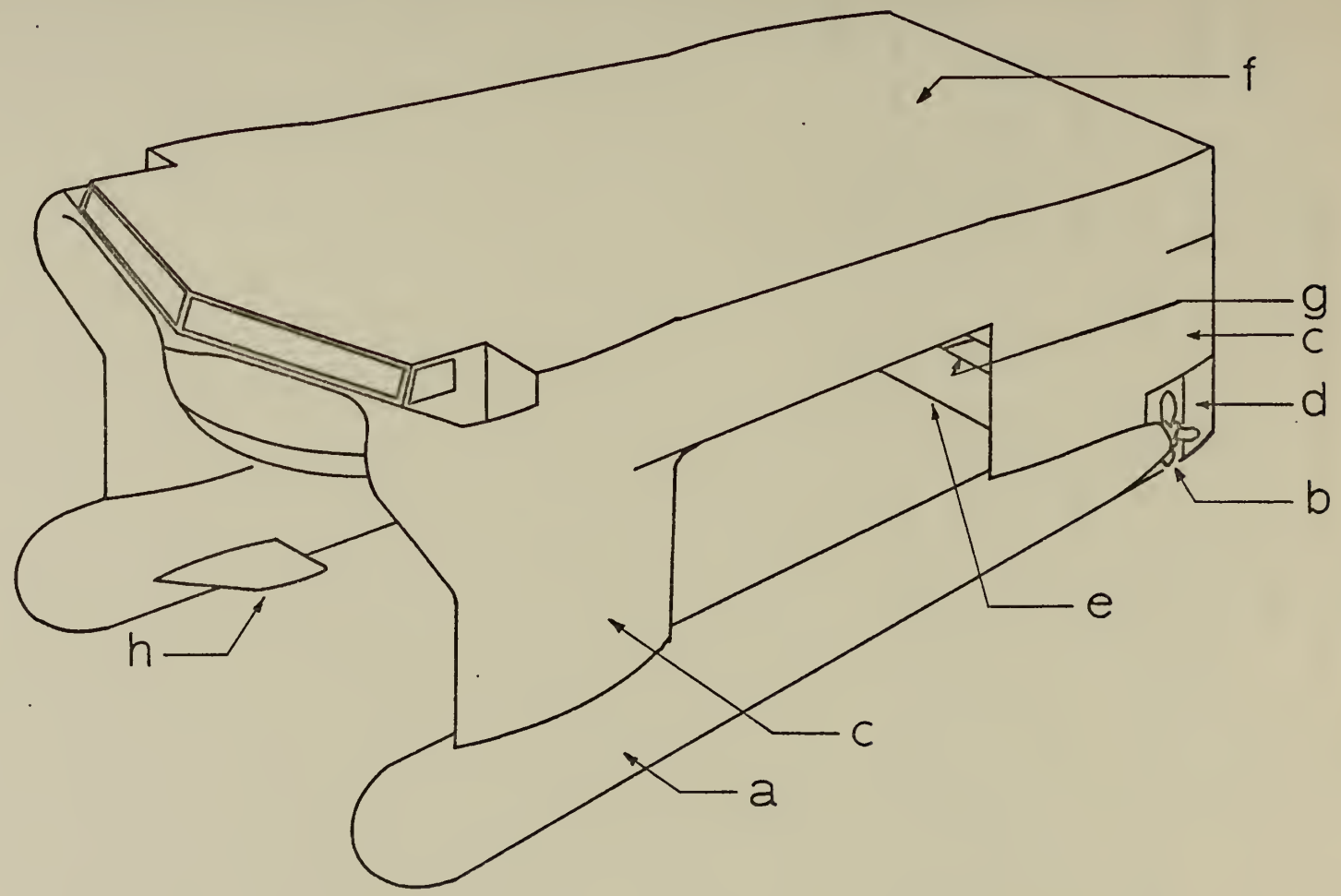

Figure 1. Semisubmerged Ship Concept.

rigid attachment of the two hull-strut systems. This rigid system provides stability in roll.

g. The elevator control surfaces incorporated in the horizontal tail surface allow a means of controlling the ship's response to waves when used in conjunction with

h. the canard control surfaces which are mounted on the forward part of the submerged hulls.

The purpose of this investigation was to consider various ways of controlling the elevator and canard surfaces and make recommendations concerning the method of control and areas requiring further investigation.

II. PROBLEM FORMULATION

A. NON-DIMENSIONALITY

The response of a ship to forces -- environmental and/or self-induced by control surface deflections -- can be determined from a system of equations which describe the ship. If the particular ship's displacement is changed, but the hull configuration is retained, it is necessary to determine a new system of equations in order to determine the ship's response. A non-dimensional system of equations however, will allow the response to be determined from this one system of equations and scaled to any particular displacement. The system of equations in this investigation are written in a non-dimensional form based on Froude number scaling.

$$\text{Froude number} = \frac{U}{\sqrt{g\ \nabla^{1/3}}} \tag{2.1}$$

where

U = ship's forward velocity

g = acceleration of gravity

∇ = ship's displacement volume

The dimensional values can be determined by multiplying the non-dimensional values by their appropriate factors as given in Table I. The symbol ρ indicates water density. The characteristic length, l_c, is the cube root of the displacement volume, ∇. The ship's displacement weight is denoted by the symbol Δ.

TABLE I

CONVERSION FACTORS

(Non-dimensional to Dimensional)

Variable	Multiplication Factor
Length	$l_c = \nabla^{1/3}$
Area	$l_c^2 = \nabla^{2/3}$
Volume	$l_c^3 = \nabla$
Linear Velocity	$\sqrt{g\ l_c}$
Time	$\sqrt{l_c/g}$
Angular Rate, Frequency	$\sqrt{g/l_c}$
Linear Acceleration	g
Angular Acceleration	g/l_c
Force	$\rho g \nabla = \Delta$

B. SEMISUBMERGED SHIP FORM USED FOR THEORETICAL ANALYSIS

The geometric form used in this discussion and in investigations conducted by Naval Undersea Center (NUC), San Diego, California, is shown in Fig. 2. The dimensions are given in terms of the characteristic length, l_c. Figure 3 will aid in interpreting the results for a specific ship size.

C. COORDINATE SYSTEM

A body moving in a fluid can move in all six degrees of freedom of motion -- translation along each of three orthogonal

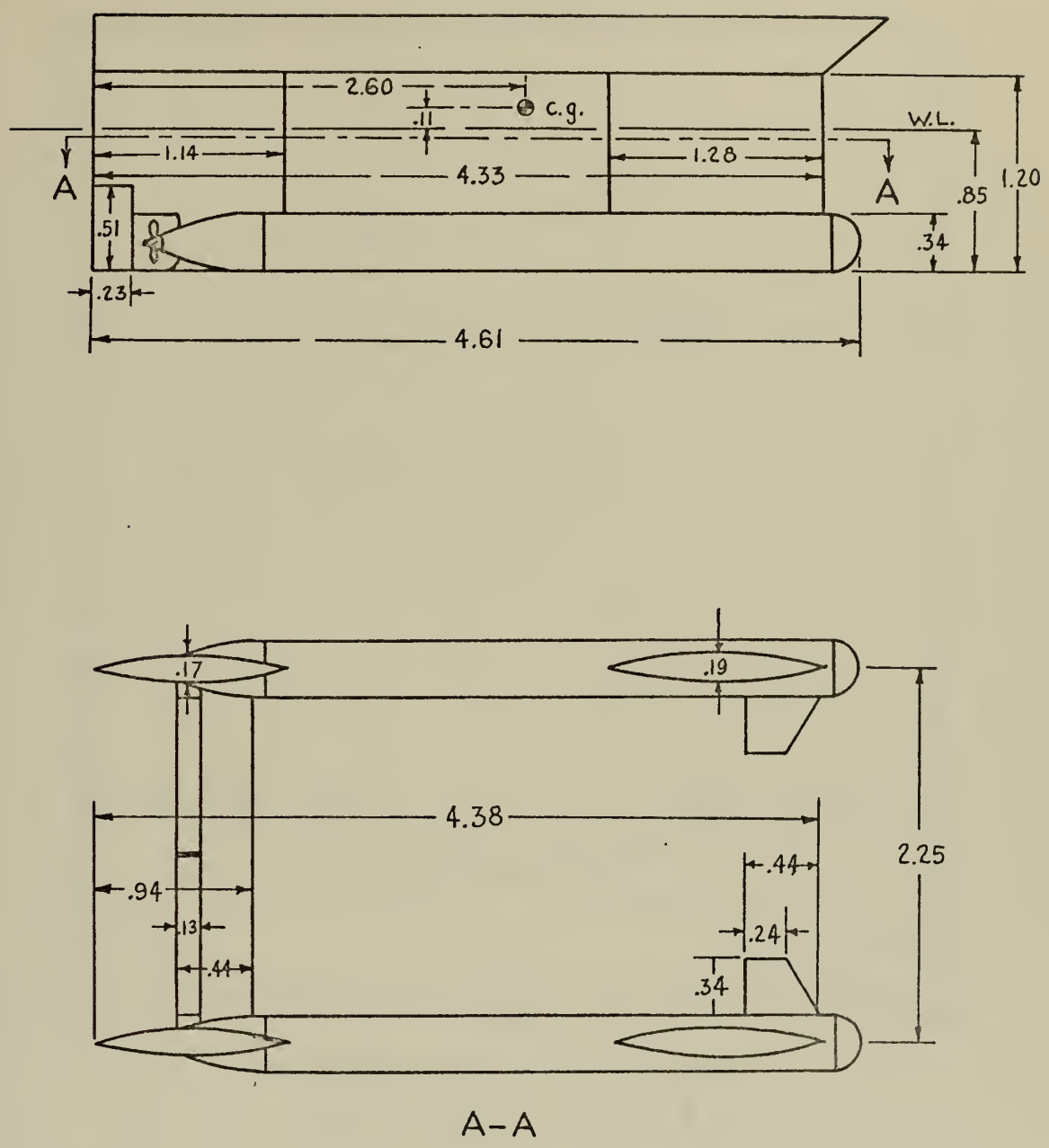

Figure 2. Semisubmerged Ship Form Used for Theoretical Analysis [Ref. 3].

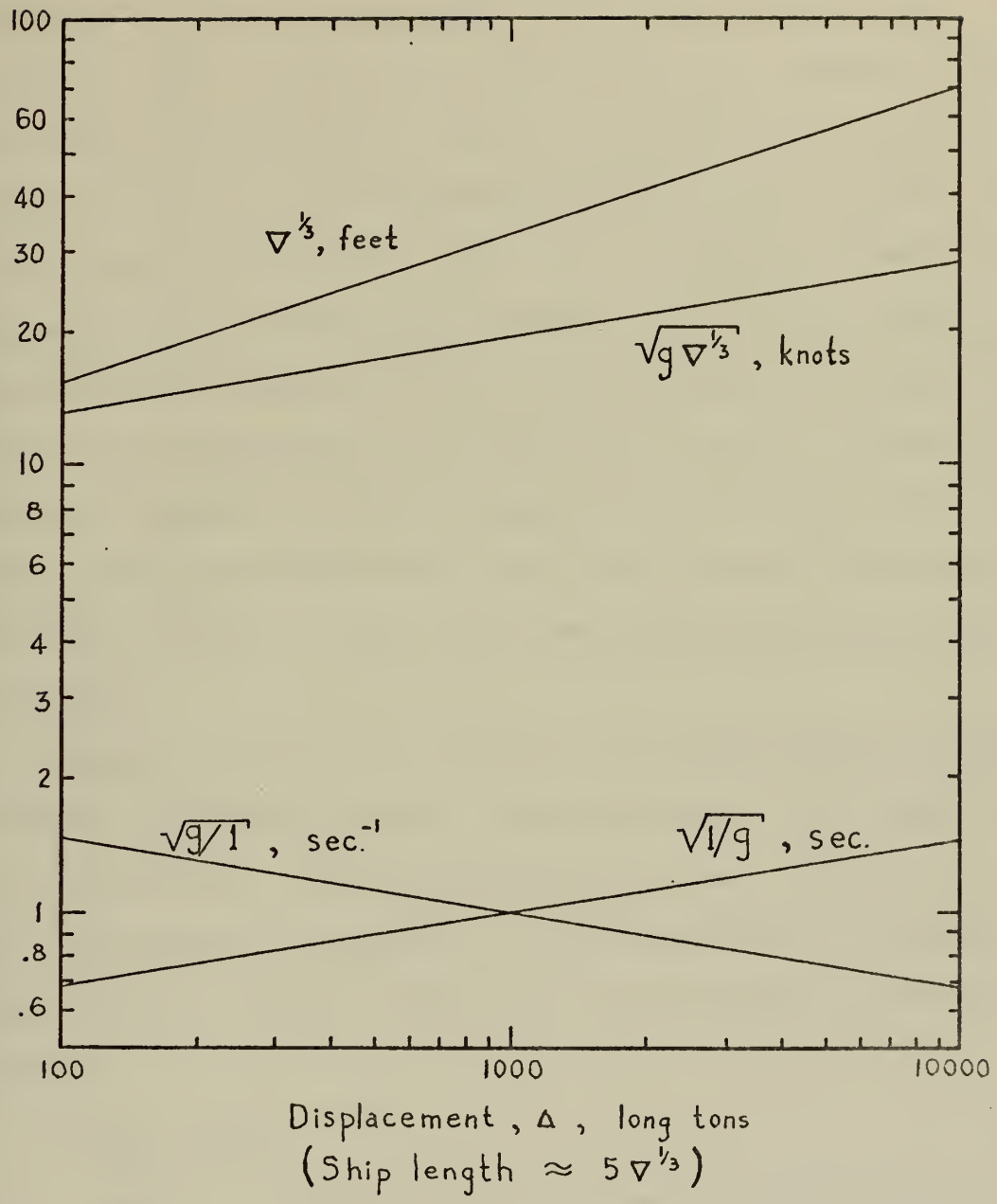

Figure 3. Basic Scaling Parameters as a Function of Ship Displacement [Ref. 3].

axes and rotation about each of the three axes. The choice of a coordinate system should result in one in which the equations of motion can be most conveniently analyzed. The ship under investigation has a plane of symmetry (as have most ships in existence), which is defined by the ship's centerline and a line perpendicular to the ship's deck. Two of the axes are chosen to lie in this plane of symmetry with the remaining axis being orthogonal to the plane. This choice simplifies the expressions for the hydrodynamic forces through symmetry and simplifies the equations of motion because the axes are parallel to the principal axes of inertia [Ref. 2]. If the axes are located such that their origin coincides with the ship's center of mass, the axes become the principal axes of inertia.

The ship's position is given in an Earth-fixed right-hand cartesian coordinate system. The attitude of the ship can be given by an Euler angle transformation from axes parallel to the Earth-fixed axis system to the ship-fixed axis system as chosen above. Figure 4 shows the relationship between the two systems.

1. X_s - Axis

The X_s - axis is the ship-fixed longitudinal axis in the plane of symmetry with the positive direction being toward the bow. It is parallel to the ship's keel.

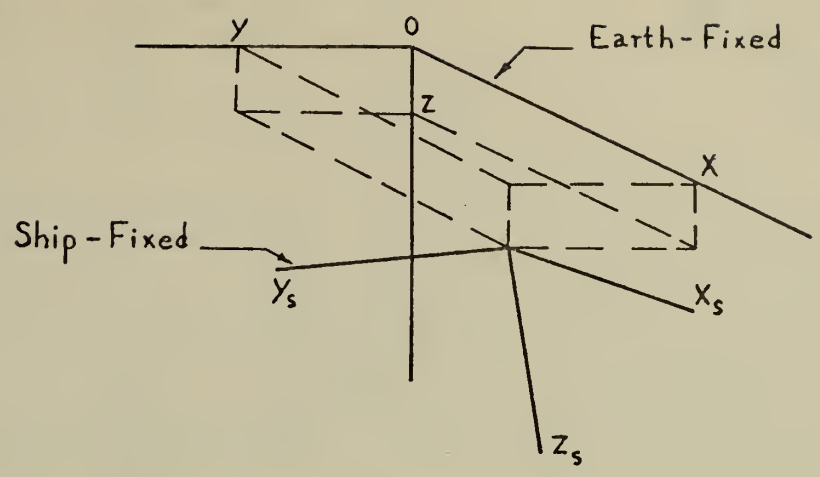

Figure 4. Coordinate System

2. Y_s - Axis

The Y_s - axis is the ship-fixed transverse axis perpendicular to the plane of symmetry. The positive direction is measured to starboard.

3. Z_s - Axis

The Z_s - axis is the "vertical axis" in the plane of symmetry. The positive direction is measured downward toward the keel.

4. Angles of Rotation

The axis system that has been chosen forms a consistent right-handed coordinate system. If ϕ is the roll angle, θ is the pitch angle and ψ is the yaw angle, then the positive rotations are as indicated in Fig. 5.

D. LONGITUDINAL RESPONSE PROBLEM

Previous investigations of the semisubmerged ship response [Ref. 3] have indicated that the pitch and heave motions in

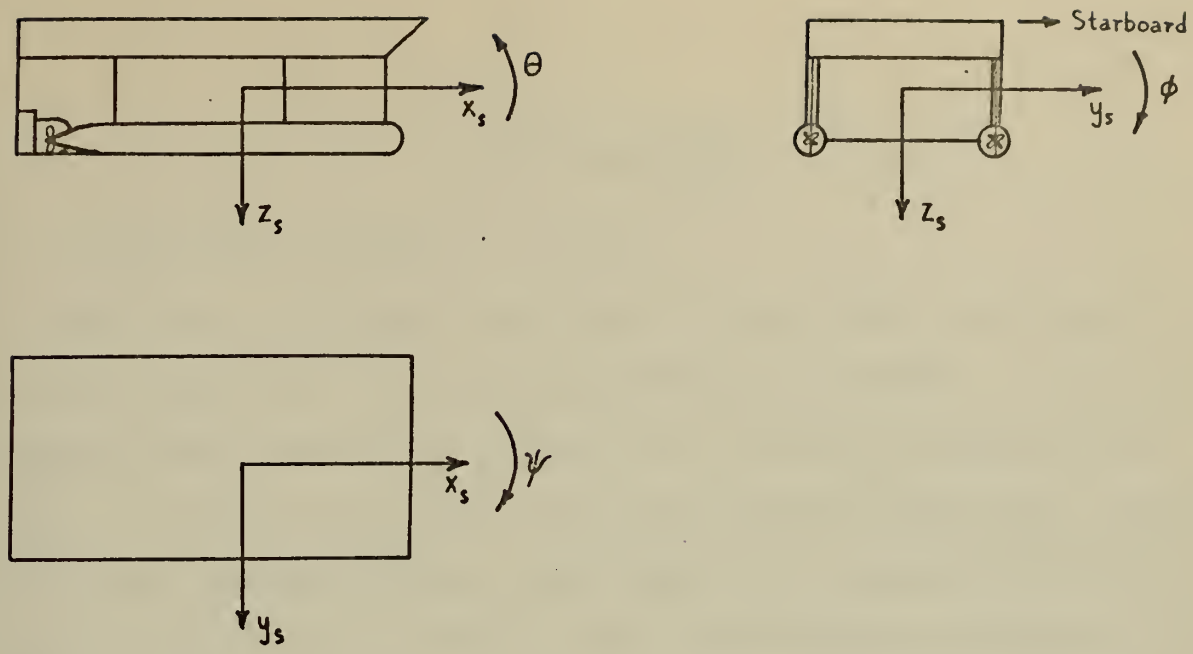

Figure 5. Angles of Rotation

regular head waves are quite small. However, the ship's natural longitudinal response to following waves having a height-to-wavelength ratio of 0.05 has been found to be greater than desired for wavelengths greater than one ship length. In particular, with a non-dimensional speed of 1.65 (high speed), the heave response to a wavelength of 10 (height of 0.5) has been found to be greater than 1.25. The values for a 2000-ton-displacement ship (refer to Fig. 2 for dimensions) would be

$$\text{wavelength, } \lambda = 410 \text{ ft.}$$

$$\text{wave height, } h = 20.5 \text{ ft.}$$

$$\text{heave, } z = 51.25 \text{ ft.}$$

This indicates that the ship is thrown out of the water, then slammed back into the water with the upper platform submerging. This is an intolerable situation and calls for the design of a control system to reduce the ship's natural response.

III. SIMULATION MODEL

A. LINEARIZED EQUATIONS OF MOTION

The force and moment equations for the ship are written in the ship-fixed axis system previously described. If it is assumed that the ship is running at a constant forward velocity, U, and that all other motion variables (velocities, angular rates and attitude angles) make only small perturbations about straight equilibrium running, then the equations of motion can be linearized and separated into two groups: (1) the lateral-direction equations, and (2) the longitudinal-direction equations [Ref. 2].

The longitudinal-direction equations of motion [Ref. 3] are written below in non-dimensional form. The vertical force is given by Eq. (3.1), the pitching moment by Eq. (3.2) and the kinematic relationships by Eqs. (3.3). The notation is defined in Table II.

$$\begin{aligned}\hat{w}'-\hat{U}\hat{q} = {} & 1/2\,\hat{U}^2[C_{z_{\hat{w}}}\hat{w}+C_{z_{\hat{q}}}\hat{q} \\ & +C_{z_{\delta_e}}\delta_e+C_{z_{\delta_c}}\delta_c] - \hat{A}_{wL}\hat{z} \\ & +\hat{A}_{wL}\hat{\bar{X}}_{wL}\theta+\hat{Z}_{\hat{w}'}\hat{w}' \\ & +\hat{Z}_{\hat{q}'}\hat{q}' + \hat{Z}_w\end{aligned} \tag{3.1}$$

TABLE II

DEFINITION OF NOTATION FOR EQUATIONS OF MOTION

C_m	$M/(1/2\,\rho U^2 \nabla)$
$\hat{Z}_w$	Non-dimensional wave forces along the X_s axis
$\hat{M}_w$	Non-dimensional wave pitching moment about the Y_s axis
$\hat{A}_{wL}$	Non-dimensional water plane area
$\hat{\bar{X}}_{wL}$	Non-dimensional X_s coordinate of the centroid of water plane area
$\hat{z}_{mp}$	Non-dimensional metacentric height in pitch
$\hat{\rho}_y$	Non-dimensional radius of gyration about the pitch axis, Y_s
θ	Pitch angle
$\hat{z}$	Non-dimensional position of ship center of mass in Earth-fixed coordinates
δ_e , δ_c	Elevator and canard surface deflection angles
U , w	Velocity component along X_s and Z_s axes respectively
q	Angular rate about Y_s axis
Z	Force along Z_s axis
M	Moment about Y_s axis
$\hat{\square}$	Non-dimensional form of $\square$
τ	Non-dimensional time, $t(g/\nabla^{1/3})^{1/2}$
$\square'$	$d\square/d\tau$
$\hat{Z}_\square$, $\hat{M}_\square$	$\frac{\partial}{\partial \square}\hat{Z}$, $\frac{\partial}{\partial \square}\hat{M}$
C_z	$Z/(1/2\,\rho U^2 \nabla^{2/3})$
$C_{z_\square}$	$\frac{\partial}{\partial \square} C_z$

$$\rho_y^2\hat{q}' = 1/2\ \hat{U}^2[C_{m_{\hat{w}}}\hat{w}+C_{m_{\hat{q}}}\hat{q}$$

$$+C_{m_{\delta_e}}\delta e+C_{m_{\delta_c}}\delta_c] - z_{mp}\theta$$

$$+\hat{A}_{wL}\hat{\bar{X}}_{wL}\hat{z} + \hat{M}_{\hat{w}'}\hat{w}' \tag{3.2}$$

$$+\hat{M}_{\hat{q}'}\hat{q}' + \hat{M}_w$$

$$\theta' = \hat{q}$$

$$\hat{z}' = -\hat{U}\theta + \hat{w} \tag{3.3}$$

The equations of motion can be rewritten in a form where they are functions of pitch (θ), heave ($\hat{z}$), their respective first and second derivatives, control surface deflections and wave forces and moments by utilizing Eqs. (3.3). These equations are shown below with the coefficients replaced by their numerical values as determined by previous investigations [Ref. 3].

$$\hat{z}'' = 1/2\ \hat{U}[-4.303(\hat{z}'+\hat{U}\theta)$$

$$-4.763\theta'-1.50\hat{U}\ \delta_e$$

$$-0.57\hat{U}\ \delta_c]-0.608\hat{z} \tag{3.4}$$

$$-0.0647\theta-1.075(\hat{z}''+\hat{U}\theta')$$

$$-0.364\theta'' + \hat{Z}_w$$

$$1.412\theta'' = 1/2\ \hat{U}[-4.763(\hat{z}+\hat{U}\theta)$$

$$-15.08\theta'-2.85\hat{U}\ \delta_e$$

$$+0.90\hat{U}\ \delta_c]-1.045\theta \tag{3.5}$$

$$-0.0647\hat{z}-0.364(\hat{z}''+\hat{U}\theta')$$

$$-2.142\theta''+\hat{M}_w$$

B. TRANSFER FUNCTIONS

Reference [3] gives the response of the ship in pitch and heave resulting from 0.1-radian control surface deflections. These responses can be determined from the transfer functions relating the responses to the control surface deflections. In order to determine if the model used in this investigation was equivalent to the one used by NUC, San Diego, the transfer functions were derived as shown below and the responses to the control surface deflections were obtained.

The linear Eqs. (3.4) and (3.5) can be Laplace transformed; the result of doing this is shown below with the assumption made that the initial conditions are zero. The numerical values of the coefficients a_i and b_i, i=1,...,8, are given in Table III.

$$\begin{aligned}\hat{z}(s)s^2 &= a_1\hat{z}(s)s+a_2\hat{z}(s)+a_3\theta(s)s^2 \\ &\quad +a_4\theta(s)s+a_5\theta(s)+a_6\delta_e \\ &\quad +a_7\delta_c+a_8\hat{Z}_w\end{aligned} \tag{3.6}$$

$$\begin{aligned}\theta(s)s^2 &= b_1\theta(s)s+b_2\theta(s)+b_3\hat{z}(s)s^2 \\ &\quad +b_4\hat{z}(s)s+b_5\hat{z}(s)+b_6\delta_e \\ &\quad +b_7\delta_c+b_8\hat{M}_w\end{aligned} \tag{3.7}$$

One method of determining transfer functions is to draw the signal flow graph corresponding to Eqs. (3.6) and (3.7) and then apply Mason's Gain Rule to the resulting graph [Ref. 4]. Referring to Fig. 6 and Table III for the appropriate coefficient values, it can be seen that the transfer functions will have the

TABLE III

Coefficient	Value	Coefficient	Value
a_1	$-1.0369\ \hat{U}$	b_1	$-2.2240\ \hat{U}$
a_2	-0.2930	b_2	$-0.6701\ \hat{U}^2-0.2940$
a_3	-0.1754	b_3	-0.1024
a_4	$-1.6658\ \hat{U}$	b_4	$-0.6701\ \hat{U}$
a_5	$-1.0369\ \hat{U}^2-0.0312$	b_5	-0.0182
a_6	$-0.3614\ \hat{U}^2$	b_6	$-0.4010\ \hat{U}^2$
a_7	$-0.1373\ \hat{U}^2$	b_7	$+0.1266\ \hat{U}^2$
a_8	$+0.4819$	b_8	$+0.2814$

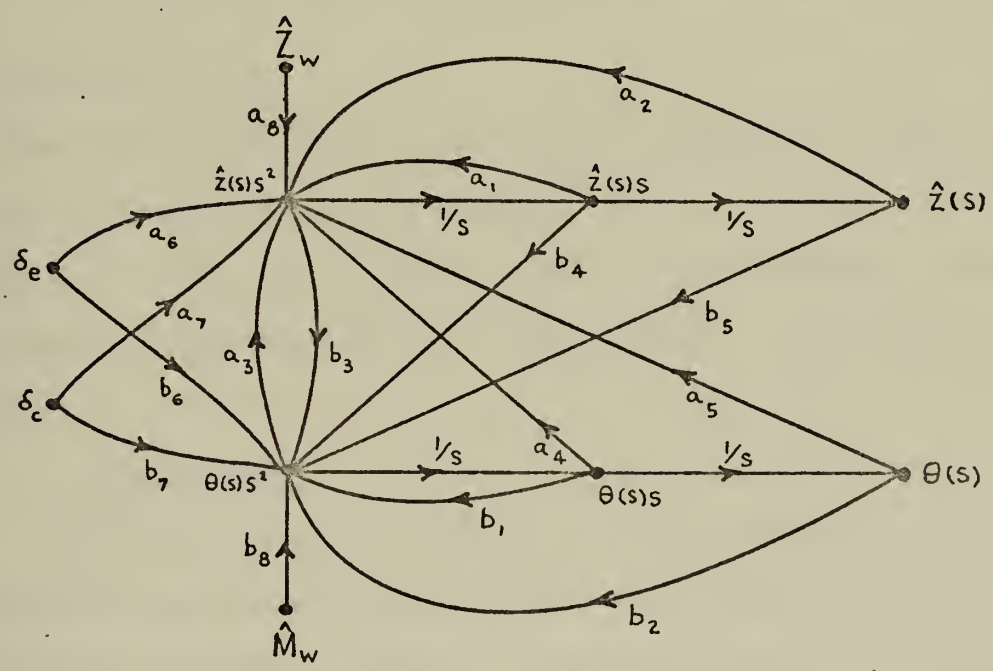

Figure 6. Signal Flow Graph.

ship's non-dimensional speed, U, as the determining parameter. The resulting transfer functions are of the form

$$\frac{X(s)}{Y(s)} = \frac{N_2 s^2 + N_1 s + N_0}{D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0} \tag{3.8}$$

where the coefficients N_i, i=0,1,2, and D_j, j=0,1,...,4 are given in Table IV.

Reference [3] investigates the ship's response for two distinct speeds, $\hat{U}$=1.0 and 1.65. Referring to Fig. 3, it can be seen that for a 2000-ton-displacement ship, these values correspond to speeds of 21.8 knots and 36 knots respectively. The transfer functions for these two speeds are given in Table V, and the responses resulting from 0.1-radian control surface deflections were found to agree with those contained in Ref. [3].

C. COMPUTER MODEL

The time histories of the ship's responses, pitch, pitch velocity, heave and heave velocity, can be found by using a computer. A choice, however, must be made whether to use analog or digital simulation for the ship model. Both models are given below.

1. Analog Model

An appropriate model for analog simulation can be easily achieved by normalizing Eq. (3.8) with repect to D_4. The resulting model is shown in Fig. 7. A penalty must be paid for the ease with which simulation has been achieved and that is that the states x_1 through x_4 have no physical significance. In order to determine some physical quantity (e.g., $\hat{z}$, $\hat{z}'$, θ, θ'), linear combinations of the states x_1 through x_4 are required.

TABLE IV

COEFFICIENT VALUES FOR EQUATION (3.8)

$\frac{X(s)}{Y(s)}$	N_2	N_1	N_0	D_4	D_3	D_2	D_1	D_0
$\frac{\theta}{\delta_c}$	$b_7+a_7b_3$	$a_7b_4-a_1b_7$	$a_7b_5-a_2b_7$	$1.0-a_3b_3$	$-a_1-b_1-a_3b_4$ $-a_4b_3$	$a_1b_1-a_2-a_3b_5$ $-b_2-a_5b_3-a_4b_4$	$a_1b_2+a_2b_1$ $-a_4b_5-a_5b_4$	$a_2b_2-a_5b_5$
$\frac{\theta}{\delta_e}$	$b_6+a_6b_3$	$a_6b_4-a_1b_6$	$a_6b_5-a_2b_6$					
$\frac{\hat{z}}{\delta_c}$	$a_7+a_3b_7$	$a_4b_7-a_7b_1$	$a_5b_7-a_7b_2$					
$\frac{\hat{z}}{\delta_e}$	$a_6+a_3b_6$	$a_4b_6-a_6b_1$	$a_5b_6-a_6b_2$					

TABLE V

TRANSFER FUNCTIONS

($\hat{U}$ = 1.0 and 1.65)

$\frac{X(s)}{Y(s)}$	$\hat{U} = 1.0$	$\hat{U} = 1.65$
$\frac{\theta}{\delta_c}$	$\frac{0.5348s^2+0.8489s+0.1505}{\beta}$	$\frac{0.6734s^2+1.7639s+0.1896}{\beta}$
$\frac{\theta}{\delta_e}$	$\frac{-1.383s^2-0.6597s-0.4216}{\beta}$	$\frac{-1.7421s^2-1.3707s-0.5309}{\beta}$
$\frac{\hat{Z}}{\delta_c}$	$\frac{-0.6066s^2-1.963s-1.017}{\beta}$	$\frac{-0.7638s^2-4.0785s-3.1226}{\beta}$
$\frac{\hat{Z}}{\delta_e}$	$\frac{-1.1066s^2-0.5168s+0.3032}{\beta}$	$\frac{-1.3935s^2-1.0737s+1.8127}{\beta}$
β	$3.733s^4+11.301s^3+8.874s^2+3.441s+1.0$	$1.727s^4+8.624s^3+9.415s^2+2.676s+1.0$

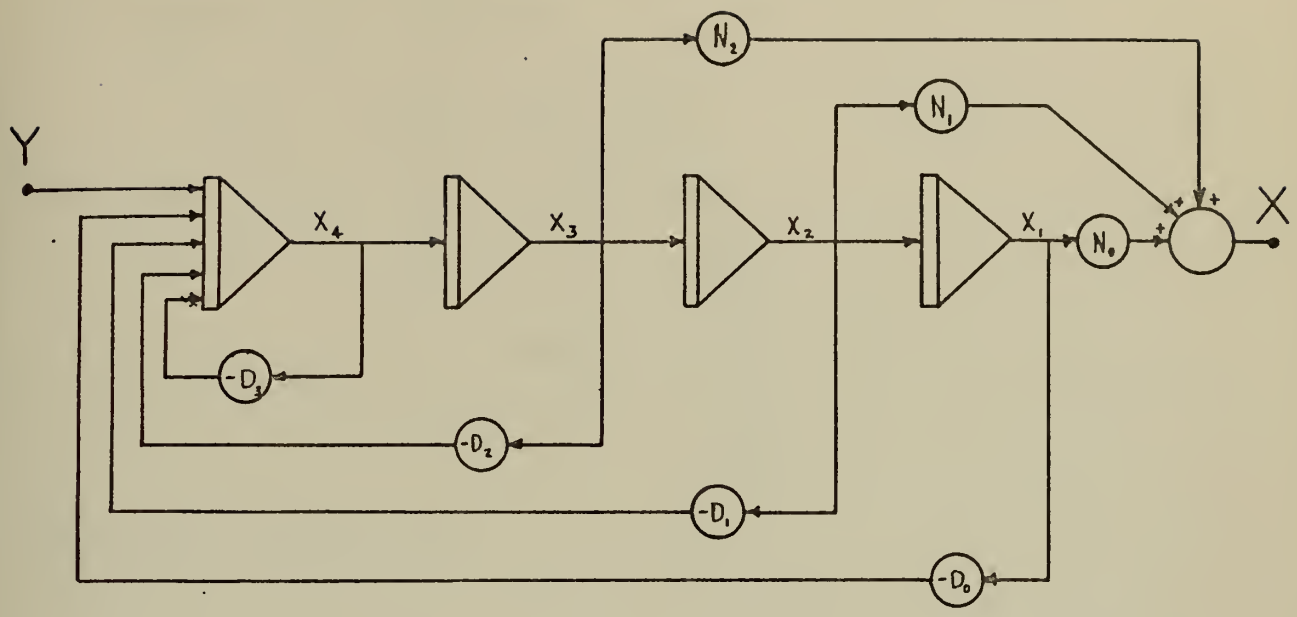

Figure 7. Analog Model

(Assumes no phase reversal through integrators)

2. Digital Model

The digital model could easily be solved by choosing the states as being the same as those in the analog model, again, however, the chosen states have no physical meaning.

A more desirable model is achieved by choosing the states as being heave, heave velocity, pitch and pitch velocity. These states can be obtained by algebraic manipulation of Eqs. (3.6) and (3.7). In their present form, each of these equations contains the transforms of the second derivative of both heave and pitch. This explicit interdependence, illustrated by the a_3-b_3 loop in Fig. 6, must be removed in order to obtain a workable model. This is accomplished by substituting Eq. (3.6) into (3.7) and Eq. (3.7) into (3.6).

The states are chosen as indicated above, resulting in a state differential equation which describes the ship. This equation has the form

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t)+\underline{B}\underline{u}(t)+\underline{C}\underline{w}(t) \tag{3.9}$$

where the state vector is defined as

$$\underline{x}(t) \equiv \begin{bmatrix} \hat{z} \\ \dot{\hat{z}} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

the control vector is

$$\underline{u}(t) \equiv \begin{bmatrix} \delta_e(t) \\ \\ \delta_c(t) \end{bmatrix}$$

and the wave-force vector is

$$\underline{w}(t) \equiv \begin{bmatrix} \hat{Z}_w \\ \\ \hat{M}_w \end{bmatrix}.$$

The coefficient matrices are

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.2951 & -0.9362\hat{U} & -0.9362\hat{U}^2+0.0207 & -1.299\hat{U} \\ 0 & 0 & 0 & 1 \\ 0.012 & -0.5742\hat{U} & -0.5742\hat{U}^2-0.2961 & -2.091\hat{U} \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 & 0 \\ -0.2964\hat{U}^2 & -0.1624\hat{U}^2 \\ 0 & 0 \\ -0.3706\hat{U}^2 & 0.1432\hat{U}^2 \end{bmatrix}$$

and

$$\underline{C} = \begin{bmatrix} 0 & 0 \\ 0.4907 & -0.0503 \\ 0 & 0 \\ -0.0502 & 0.2865 \end{bmatrix}$$

Equation (3.9) is readily simulated on a digital computer to give the ship's time response to control surface deflections and known wave forces and moments.

IV. CONTROL SYSTEMS FOR THE SEMISUBMERGED SHIP

Optimal control theory [Ref. 5] was used to derive three controllers for the semisubmerged ship. The derivations of the control laws are based upon two assumptions: (1) the states in Eq. (3.9) are directly measurable, and (2) no measurement noise exists in the measurement of the states.

A. PERFORMANCE MEASURE

The performance measure gives a quantitative measure of the quality of response of the system under consideration. With the exception of long waves ($\hat{\lambda}>14$) where the wave crest might impact with the upper platform and it would be more advantageous to follow the contour of the wave, it is desired to maintain the ship as stable as possible in the Earth-fixed coordinate system. This requires that the state vector be maintained close to the equilibrium point (origin). The control surfaces however, should not have excessive deflections which would run them into mechanical or electrical stops, nor should they overload the actuation system and result in loss of the control system. In view of the above, the requirement for the control system becomes:

> The control system must maintain the state vector, $\underline{x}(t)$, as close to the origin as possible without an excessive expenditure of control effort.

The performance measure then chosen is

$$J = 1/2 \int_{t_0}^{t_f} [\underline{x}^T(t)\underline{\underline{Q}}\underline{x}(t)+\underline{u}^T(t)\underline{\underline{R}}\underline{u}(t)]dt \qquad (4.1)$$

where the final time t_f is assumed to be fixed, $\underline{Q}$ is a real symmetric positive semi-definite weighting matrix and $\underline{R}$ is a real symmetric positive definite weighting matrix. It is assumed that the states and controls are not bounded and that the final state, $\underline{x}(t_f)$, is free. No penalty has been placed on minimizing the deviation of the final state from its desired value, $\underline{0}$. It is desired to determine the control $\underline{u}^*(t)$ that minimizes this performance measure.

B. CONTROL LAW FOR THE SHIP IN A CALM SEA

The system of equations defining the ship in a calm sea ($\underline{w}(t)=\underline{0}$) is

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}+\underline{B}\underline{u}(t) \tag{4.2}$$

The requirement to minimize the performance measure J, given by Eq. (4.1) leads to the well-known optimal control law [Ref. 5]

$$\begin{aligned} \underline{u}^*(t) &= -\underline{R}^{-1}\underline{B}^T\underline{K}(t)\underline{x}(t) \\ &\equiv \underline{F}(t)\underline{x}(t) \end{aligned} \tag{4.3}$$

where $\underline{K}(t)$ is the real symmetric positive definite solution to the Riccati equation

$$\dot{\underline{K}}(t) = -\underline{K}(t)\underline{A}-\underline{A}^T\underline{K}(t)-\underline{Q}+\underline{K}(t)\underline{B}\underline{R}^{-1}\underline{B}^T\underline{K}(t) \tag{4.4}$$

which satisfies the boundary condition

$$\underline{K}(t_f) = \underline{0}$$

The optimal control law given in Eq. (4.3) is seen to be a time-varying feedback of the state vector $\underline{x}(t)$. The closed-loop system state equations reduce to

$$\dot{\underline{x}}(t) = [\underline{A}+\underline{B}\underline{F}(t)]\underline{x}(t) \tag{4.5}$$

when the optimal control law is used. Figure 8 shows this system in block diagram form.

C. THE OPTIMAL CONTROL LAW FOR THE SHIP WHEN SUBJECTED TO WAVE FORCES

The Hamiltonian for the forced system is

$$\begin{aligned}(\underline{x}(t),\underline{u}(t),\underline{p}(t),t) &= 1/2\underline{x}^T(t)\underline{Q}\underline{x}(t)+1/2\ \underline{u}^T(t)\underline{R}\underline{u}(t)\\ &\quad +\underline{p}^T(t)\underline{A}\underline{x}(t)+\underline{p}^T(t)\underline{B}\underline{u}(t)\\ &\quad +\underline{p}^T(t)\underline{C}\underline{w}(t)\end{aligned} \tag{4.6}$$

where $\underline{p}(t)$ is a vector of Lagrange multipliers called costates. The theorem that the variation of the augmented functional

$$\begin{aligned}J_a &= \int_{t_0}^{t_f}\{1/2[\underline{x}^T(t)\underline{Q}\underline{x}(t)+\underline{u}^T(t)\underline{R}\underline{u}(t)]\\ &\quad +\underline{p}^T(t)[\underline{A}\underline{x}(t)+\underline{B}\underline{u}(t)+\underline{C}\underline{w}(t)-\dot{\underline{x}}(t)]\}dt\end{aligned} \tag{4.7}$$

equals zero on an extremal requires that the following necessary conditions hold for the state equation, costate equation, and control [Ref. 5]:

$$\left.\begin{aligned}\dot{\underline{x}}^*(t) &= Ax^*(t)+Bu^*(t)+Cw(t) && (4.8)\\ \dot{\underline{p}}^*(t) &= -\frac{\partial}{\partial\underline{x}} = -\underline{Q}\underline{x}^*(t)-\underline{A}^T\underline{p}^*(t) && (4.9)\\ \underline{0} &= -\frac{\partial}{\partial\underline{u}} = \underline{R}\underline{u}^*(t)+B^Tp^*(t) && (4.10)\end{aligned}\right\}\ \text{for all } t=[t_0,t_f]$$

where the asterisk denotes an extremal. $\underline{R}$ is a positive definite matrix, therefore Eq. (4.9) can be solved for the optimal control to give

$$\underline{u}^*(t) = -\underline{R}^{-1}\underline{B}^T\underline{p}^*(t) \qquad 4.11)$$

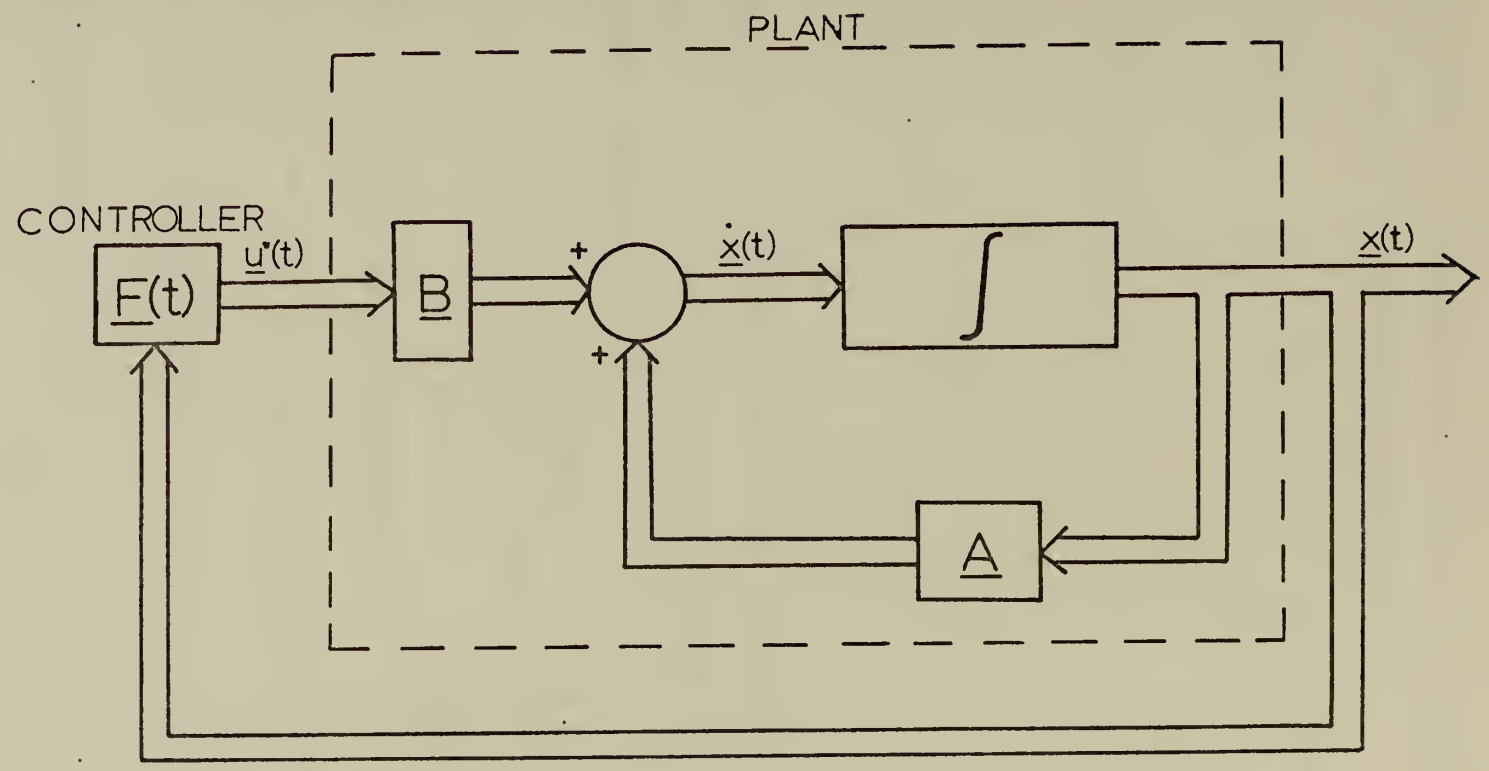

Figure 8. Linear Regulator Plant with Optimal Feedback Controller

Substitution of Eq. (4.11) into Eq. (4.8) yields

$$\dot{\underline{x}}^*(t) = \underline{A}\underline{x}^*(t) - \underline{B}\underline{R}^{-1}\underline{B}^T\underline{p}^*(t) + \underline{C}\underline{w}(t) \tag{4.12}$$

Equations (4.9) and (4.12) now form a system of 2n linear nonhomogeneous differential equations

$$\begin{bmatrix} \dot{\underline{x}}^*(t) \\ \hline \dot{\underline{p}}^*(t) \end{bmatrix} = \left[\begin{array}{c|c} \underline{A} & -\underline{B}\underline{R}^{-1}\underline{B}^T \\ \hline -\underline{Q} & -\underline{A}^T \end{array}\right] \begin{bmatrix} \underline{x}^*(t) \\ \hline \underline{p}^*(t) \end{bmatrix} + \begin{bmatrix} \underline{C}\underline{w}(t) \\ \hline \underline{0} \end{bmatrix} \tag{4.13}$$

whose solution is

$$\begin{bmatrix} \underline{x}^*(t_f) \\ \hline \underline{p}^*(t_f) \end{bmatrix} = \underline{\phi}(t_f,t) \begin{bmatrix} \underline{x}^*(t) \\ \hline \underline{p}^*(t) \end{bmatrix} + \int_t^{t_f} \underline{\phi}(t_f,\tau) \begin{bmatrix} \underline{C}\underline{w}(\tau) \\ \hline \underline{0} \end{bmatrix} d\tau \tag{4.14}$$

where $\underline{\phi}$ is the state transition matrix of Eq. (4.13). If $\underline{\phi}$ is partitioned and the integral in Eq. (4.14) is replaced by a 2nxl vector

$$\int_t^{t_f} \underline{\phi}(t_f,\tau) \begin{bmatrix} \underline{C}w(\tau) \\ \hline \underline{0} \end{bmatrix} d\tau = \begin{bmatrix} \underline{f}_1(t) \\ \hline \underline{f}_2(t) \end{bmatrix} \tag{4.15}$$

Eq. (4.14) can be rewritten as

$$\underline{x}^*(t_f) = \underline{\phi}_{11}(t_f,t)\underline{x}^*(t) + \underline{\phi}_{12}(t_f,t)\underline{p}^*(t) + \underline{f}_1(t) \tag{4.16}$$

$$\underline{p}^*(t_f) = \underline{\phi}_{21}(t_f,t)\underline{x}^*(t) + \underline{\phi}_{22}(t_f,t)\underline{p}^*(t) + \underline{f}_2(t) \tag{4.17}$$

The boundary condition for the costates is

$$\underline{p}^*(t_f) = \underline{0}$$

Therefore, Eq. (4.17) can be solved as

$$\begin{aligned} \underline{p}^*(t) &= -\underline{\phi}_{22}^{-1}(t_f,t)\underline{\phi}_{21}(t_f,t)\underline{x}^*(t) \\ &\quad -\underline{\phi}_{22}^{-1}(t_f,t)\underline{f}_2(t) \end{aligned} \tag{4.18}$$

Kalman [Ref. 6] has shown that the required inverse, $\underline{\phi}_{22}^{-1}(t_f,t)$, exists for all $t \in [t_0,t_f]$ so that Eq. (4.18) can be rewritten as

$$\underline{p}^*(t) = \underline{K}(t)\underline{x}^*(t)+\underline{s}(t) \tag{4.19}$$

Equation (4.19) can now be substituted into Eq. (4.11) resulting in the optimal control law (Eq. (4.20)) similar to that in Ref. [7].

$$\begin{aligned} \underline{u}^*(t) &= -\underline{R}^{-1}(t)\underline{B}^T(t)\underline{K}(t)\underline{x}^*(t)-\underline{R}^{-1}(t)\underline{B}^T(t)\underline{s}(t) \\ &\equiv \underline{F}(t)\underline{x}^*(t)+\underline{v}(t) \end{aligned} \tag{4.20}$$

Equation (4.20) indicates that the optimal control law contains a linear function of the system states, and also a command signal, $\underline{v}(t)$, that not only depends upon the system parameters, but also upon future values of the wave forcing function, $\underline{w}(t)$. This indicates that the optimal control law must be anticipatory in effect.

The matrix K(t) and the vector $\underline{s}(t)$ can be obtained by differentiating Eq. (4.19)

$$\dot{\underline{p}}^*(t) = \dot{\underline{K}}(t)\underline{x}^*(t)+\underline{K}(t)\dot{\underline{x}}^*(t)+\dot{\underline{s}}(t) \tag{4.21}$$

and substituting Eqs. (4.8), (4.11) and (4.19) into Eq. (4.21).

$$\dot{\underline{p}}^*(t) = \dot{\underline{K}}(t)\underline{x}^*(t)+\underline{K}(t)[\underline{A}\underline{x}^*(t)+\underline{B}\underline{u}^*(t)+\underline{C}\underline{w}(t)]+\dot{\underline{s}}(t)$$

$$= \dot{\underline{K}}(t)\underline{x}^*(t)+\underline{K}(t)[\underline{A}\underline{x}^*(t)-\underline{B}\underline{R}^{-1}\underline{B}^T\underline{p}^*(t)+\underline{C}\underline{w}(t)]+\dot{\underline{s}}(t)$$

$$= \dot{\underline{K}}(t)\underline{x}^*(t)+\underline{K}(t)[\underline{A}\underline{x}^*(t)-\underline{B}\underline{R}^{-1}\underline{B}^T(\underline{K}(t)\underline{x}^*(t)+\underline{s}(t))$$

$$+\underline{C}\underline{w}(t)]+\dot{\underline{s}}(t) \tag{4.22}$$

However, substitution of Eq. (4.19) into the costate Eq. (4.9) also results in

$$\dot{\underline{p}}^*(t) = -\underline{Q}\underline{x}^*(t)-\underline{A}^T(\underline{K}(t)\underline{x}^*(t)+\underline{s}(t)) \tag{4.23}$$

Equations (4.22) and (4.23) can be equated which, after collecting terms, yields

$$[\dot{\underline{K}}(t)+\underline{K}(t)\underline{A}-\underline{K}(t)\underline{B}\underline{R}^{-1}\underline{B}^T\underline{K}(t)+\underline{A}^T\underline{K}(t)+\underline{Q}]\underline{x}^*(t)$$

$$+[\dot{\underline{s}}(t)+\underline{A}^T\underline{s}(t)-\underline{K}(t)\underline{B}\underline{R}^{-1}\underline{B}^T\underline{s}(t)+\underline{K}(t)\underline{C}\underline{w}(t)] = \underline{0} \tag{4.24}$$

which must be satisfied for all $\underline{x}(t)$ and $\underline{w}(t)$. This results in the matrix differential equation for $\underline{K}(t)$,

$$\dot{\underline{K}}(t) = -\underline{K}(t)\underline{A}-\underline{A}^T\underline{K}(t)-\underline{Q}+\underline{K}(t)\underline{B}\underline{R}^{-1}\underline{B}^T\underline{K}(t) \tag{4.4}$$

which is the same as that given earlier for the calm sea condition, and an additional vector differential equation,

$$\dot{\underline{s}}(t) = -[\underline{A}^T-\underline{K}(t)\underline{B}\underline{R}^{-1}\underline{B}^T]\underline{s}(t)-\underline{K}(t)\underline{C}\underline{w}(t) \tag{4.25}$$

The boundary conditions to be satisfied are

$$\underline{K}(t_f) = \underline{0}$$

$$\underline{s}(t_f) = \underline{0}$$

Equations (4.4) and (4.25) can now be integrated backward in time from t_f to t_0 and the resulting solutions can be stored and substituted into Eq. (4.20) to give the optimal control law

$$\underline{u}'(t) = -\underline{R}^{-1}\underline{B}^T\underline{K}(t)\underline{x}^*(t)-\underline{R}^{-1}\underline{B}^T\underline{s}(t) \qquad (4.20)$$

$$\equiv \underline{F}(t)\underline{x}^*(t)+\underline{v}(t)$$

The controlled system, shown in Fig. 9, is now defined by the matrix differential equation

$$\dot{\underline{x}}(t) = [\underline{A}+\underline{B}\underline{F}(t)]\underline{x}(t)+\underline{B}\underline{v}(t)+\underline{C}\underline{w}(t) \qquad (4.26)$$

D. A SUBOPTIMAL CONTROL LAW FOR THE SHIP WHEN SUBJECTED TO WAVE FORCES

The optimal control law given by Eq. (4.20) requires that the command signal $\underline{v}(t)$ be determined either by (1) integrating Eq. (4.25) backwards in time assuming the forcing signal $\underline{w}(t)$ is known, or (2) storing previously computed values of $\underline{v}(t)$ in a look-up table and selecting the one which corresponds to the appropriate time history of $\underline{w}(t)$. Either of the above methods would require the use of either an existing onboard computer or a computer dedicated specifically to the control system for computation and storage. The suboptimal control system derived below was investigated to determine if this added complexity could be reduced or altogether avoided.

Referring to the system equation given by Eq. (4.26), the objective is to have the state vector, $\underline{x}(t)$, approach $\underline{0}$. This requires that $\dot{\underline{x}}(t)$ also approach $\underline{0}$. This can only occur if when $\underline{x}(t) \simeq \underline{0}$,

$$\underline{B}\underline{v}(t) = -\underline{C}\underline{w}(t) \qquad (4.27)$$

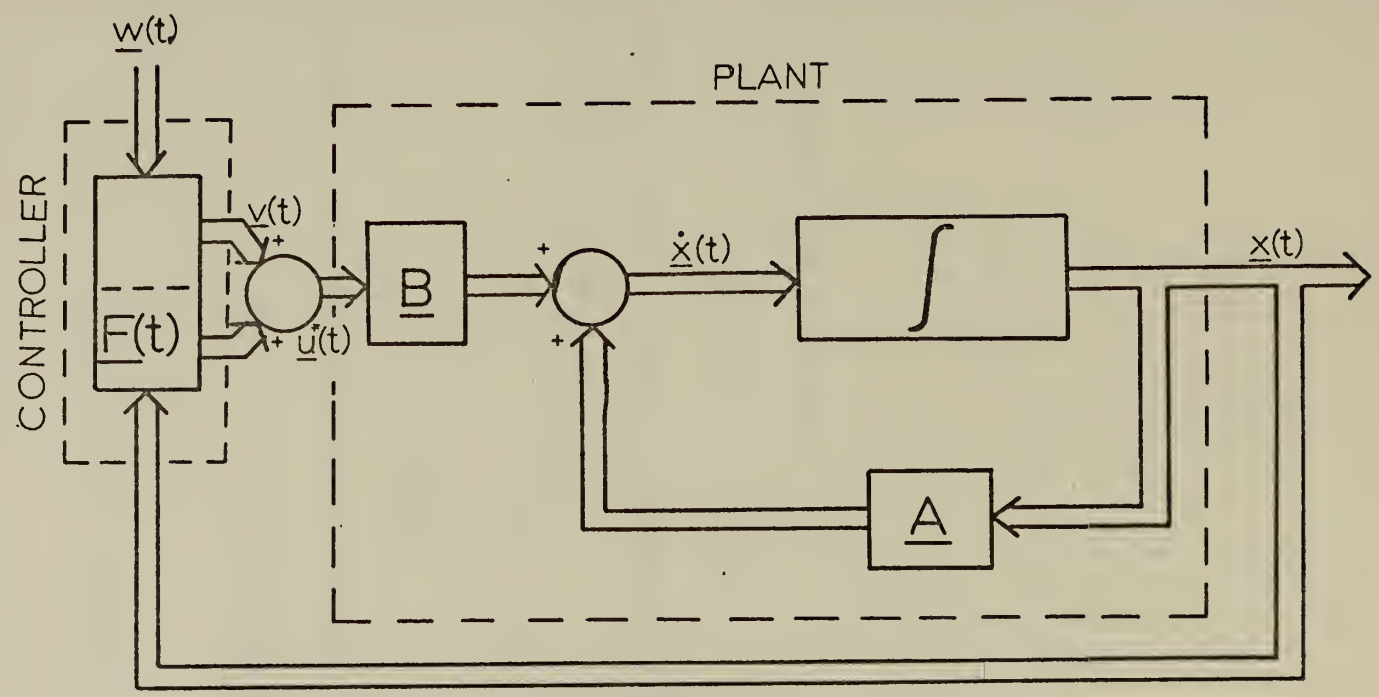

Figure 9. Linear Regulator Plant with Optimal Feedback Controller for External Disturbances (w(t)).

In general, Eq. (4.27) cannot be solved for $\underline{v}(t)$ since $\underline{B}^{-1}$ does not exist; however here the first and third rows of both $\underline{B}$ and $\underline{C}$ are zero which allows two new matrices, $\underline{B}'$ and $\underline{C}'$, to be defined as

$$\underline{B}' = \begin{bmatrix} b_{21} & b_{22} \\ b_{41} & b_{42} \end{bmatrix}$$

$$\underline{C}' = \begin{bmatrix} c_{21} & c_{22} \\ c_{41} & c_{42} \end{bmatrix} \tag{4.28}$$

It can be shown (see Appendix A) that

$$\underline{B}'\underline{v}(t) = -\underline{C}'\underline{w}(t) \tag{4.29}$$

and since $\underline{B}'$ and $\underline{C}'$ are (2x2) matrices and $\underline{B}'^{-1}$ exists, Eq. (4.29) can be solved to yield

$$\begin{aligned} \underline{v}(t) &= -\underline{B}'^{-1}\underline{C}'\underline{w}(t) \\ &\equiv \underline{G}\underline{w}(t) \end{aligned} \tag{4.30}$$

The control law can now be written as

$$\underline{u}(t) = \underline{F}(t)\underline{x}(t)+\underline{G}\underline{w}(t) \tag{4.31}$$

and Eq. (3.9) becomes

$$\dot{\underline{x}}(t) = [\underline{A}+\underline{B}\underline{F}(t)]\underline{x}(t)+[\underline{B}\underline{G}+\underline{C}]\underline{w}(t) \tag{4.32}$$

It can be easily shown that as defined, the matrix equality

$$\underline{B}\underline{G} = -\underline{C} \tag{4.33}$$

holds (see Appendix B) so that if $\underline{w}(t)$ is exactly known, Eq. (4.32) reduces to Eq. (4.5), the system equation for calm seas.

$$\dot{\underline{x}}(t) = [\underline{A}+\underline{B}\underline{F}(t)]\underline{x}(t) \tag{4.5}$$

Therefore, if the suboptimal control law given by Eq. (4.31) is used, the computer capability required by the optimal control law is avoided.

Reference [7] shows that the eigenvalues of the matrix $[\underline{A}+\underline{B}\underline{F}(t)]$ have negative real parts so that after initial transients, the system states will approach zero. Figure 10 shows the block diagram for this system.

E. CONSTANT STATE FEEDBACK

The control laws given by Eqs. (4.3), (4.20) and (4.31) contain the time-varying feedback matrix $\underline{K}(t)$. This matrix was determined, as previously stated, by integrating Eq. (4.4) backwards in time. Although $\underline{K}(t)$ can be computed and stored prior to utilization of the above control laws (since the solution of Eq. (4.4) does not depend upon the system states), the complexity of the control system would be increased over one which incorporated constant rather than time-varying feedback of the system states.

Kalman [Ref. 6] has shown that if (1) the plant is completely controllable, (2) there is no cost penalty for final state vector deviations from the desired final state, and (3) $\underline{A}$, $\underline{B}$, $\underline{R}$ and $\underline{Q}$ are constant matrices, then $\underline{K}(t)$ approaches $\underline{K}$ (a constant matrix) as t_f approaches infinity. To determine whether the

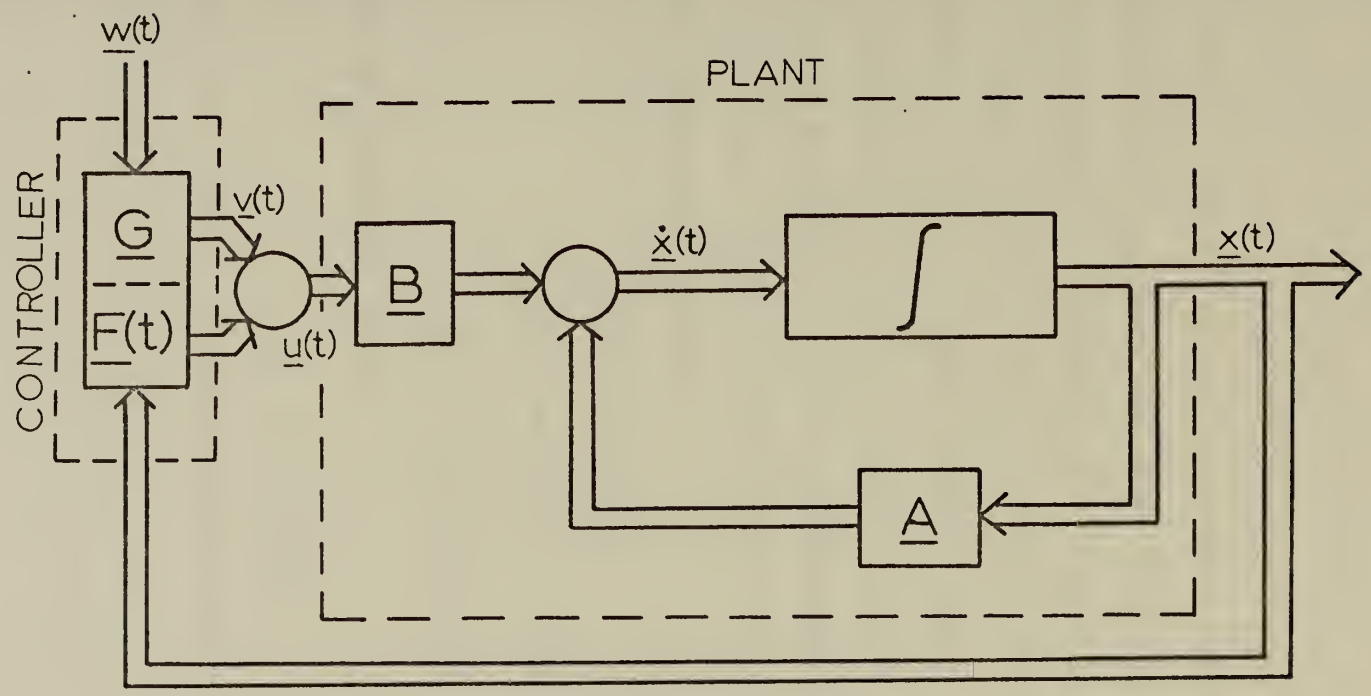

Figure 10. Linear Regulator Plant with Suboptimal Feedback Controller for External Disturbances (w(t))

plant is completely controllable, the rank of the matrix

$$\underline{E} = [\underline{B} \quad \underline{A}\underline{B} \quad \underline{A}^2\underline{B} \quad \ldots \underline{A}^{n-1}\underline{B}] \tag{4.34}$$

must be evaluated. The plant is completely controllable if and only if $\underline{E}$ has a rank of n (in this particular case n is four). Equation (4.34) was evaluated and found to have a rank of four, so this system is controllable as anticipated.

The ship meets all of the above requirements with the exception that t_f does not approach infinity (although it may be quite large). However, the solution to Eq. (4.4) using the NPS IBM-360/67 computer (Figs. 11 through 15) shows that the feedback gain matrix does reach steady-state constant values in a very short time (≃ 3.1 sec. for a 2000-ton-displacement ship) and hence $\underline{K}(t)$ can be approximated by the constant matrix $\underline{K}$ to give the following control laws which are used in the remaining investigation:

$$\underline{u}(t) = \underline{F}\underline{x}(t) \tag{4.35}$$

$$\underline{u}^*(t) = \underline{F}\underline{x}(t) + \underline{v}(t) \tag{4.36}$$

$$\underline{u}(t) = \underline{F}\underline{x}(t)+\underline{G}\underline{w}(t) \tag{4.37}$$

$\underline{F}$ can be obtained from

$$\underline{F} = -\underline{R}^{-1}\underline{B}^T\underline{K} \tag{4.38}$$

where $\underline{K}$ is the steady-state solution to Eq. (4.4) and hence also satisfies the nonlinear algebraic equation

$$\underline{0} = -\underline{K}\underline{A}-\underline{A}^T\underline{K}-\underline{Q}+\underline{K}\underline{B}\underline{R}^{-1}\underline{B}^T\underline{K} \tag{4.39}$$

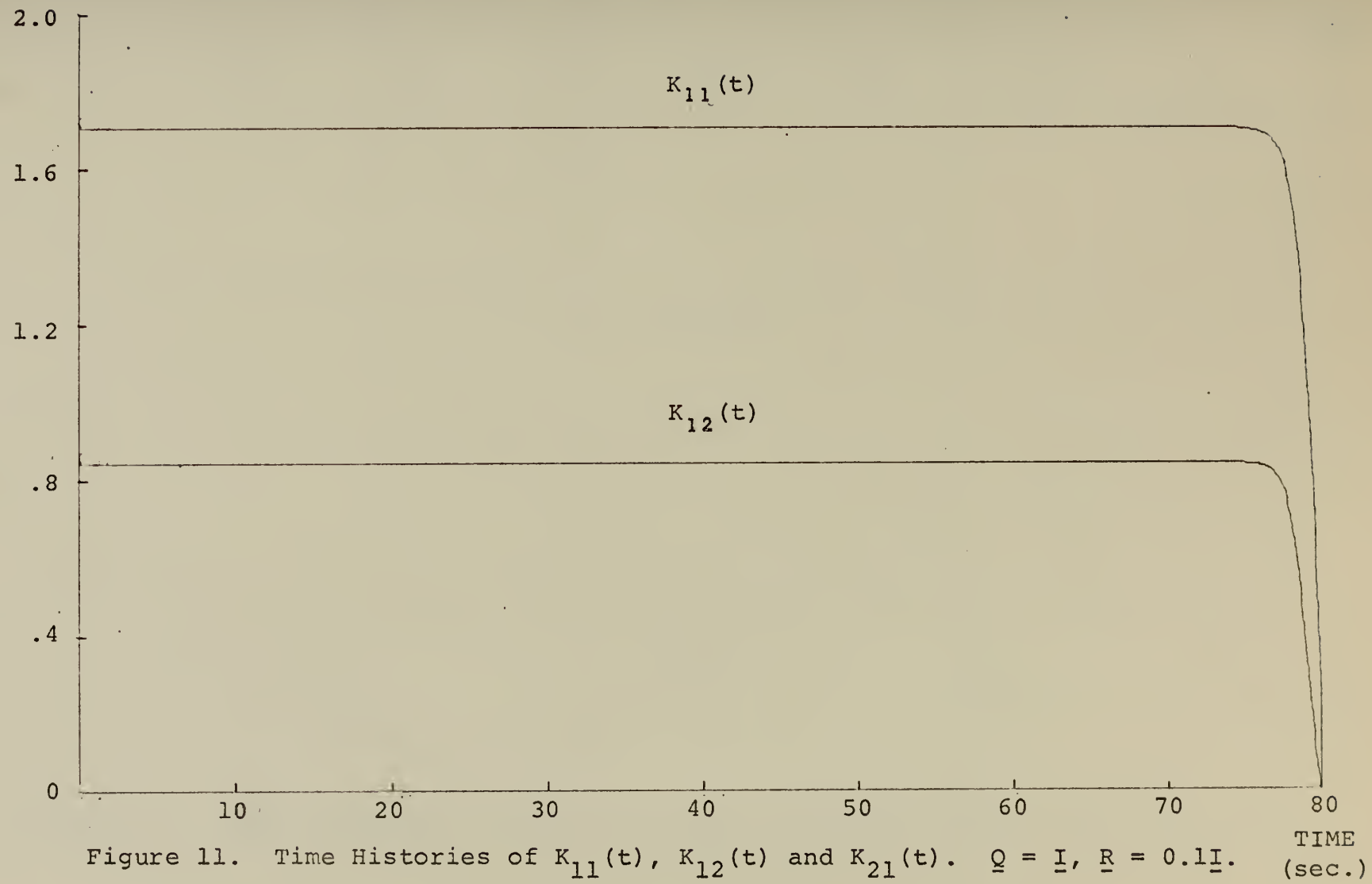

Figure 11. Time Histories of $K_{11}(t)$, $K_{12}(t)$ and $K_{21}(t)$. $\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$.

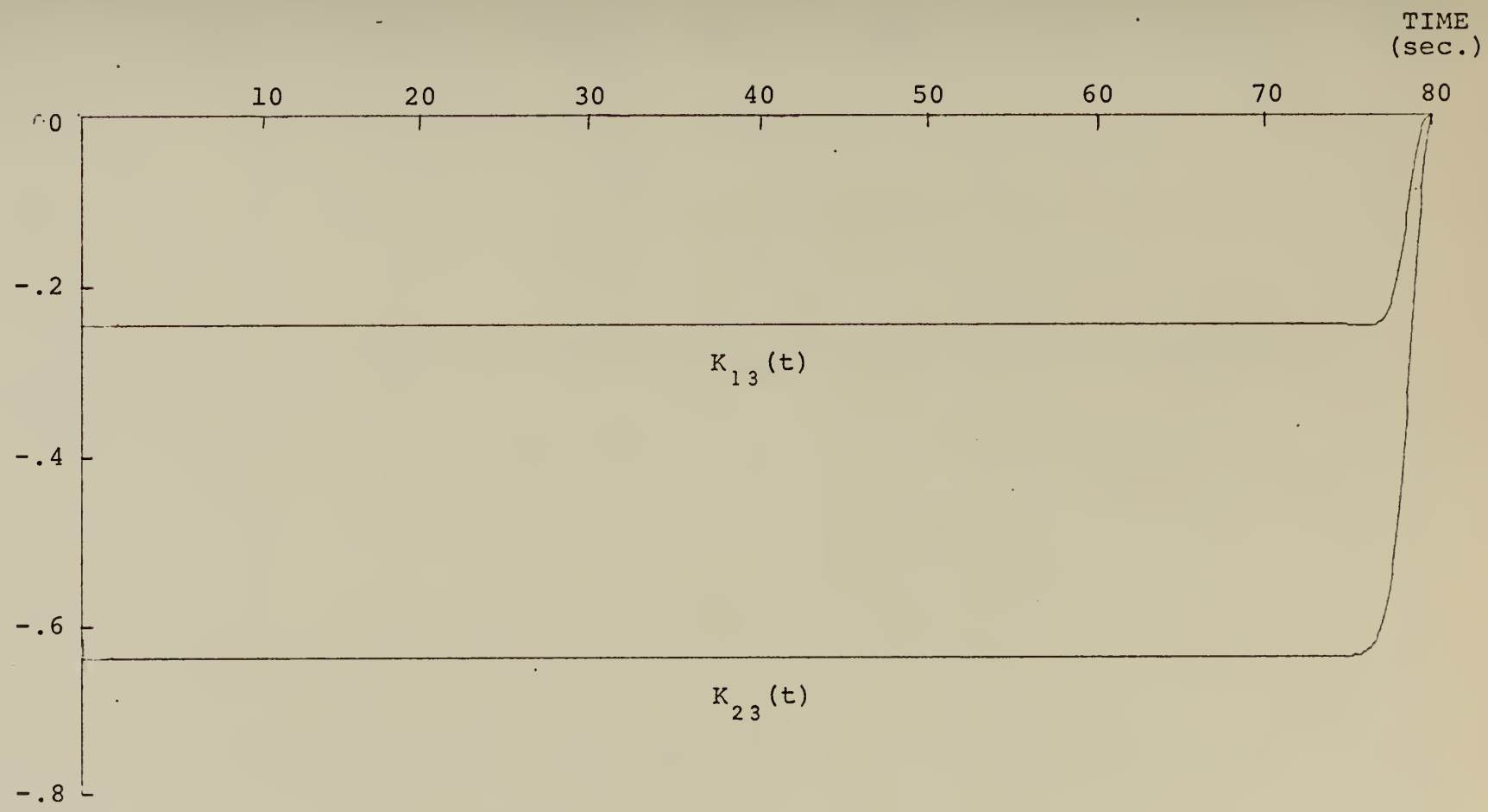

Figure 12. Time Histories of $K_{13}(t)$, $K_{23}(t)$, $K_{31}(t)$ and $K_{32}(t)$. $\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$.

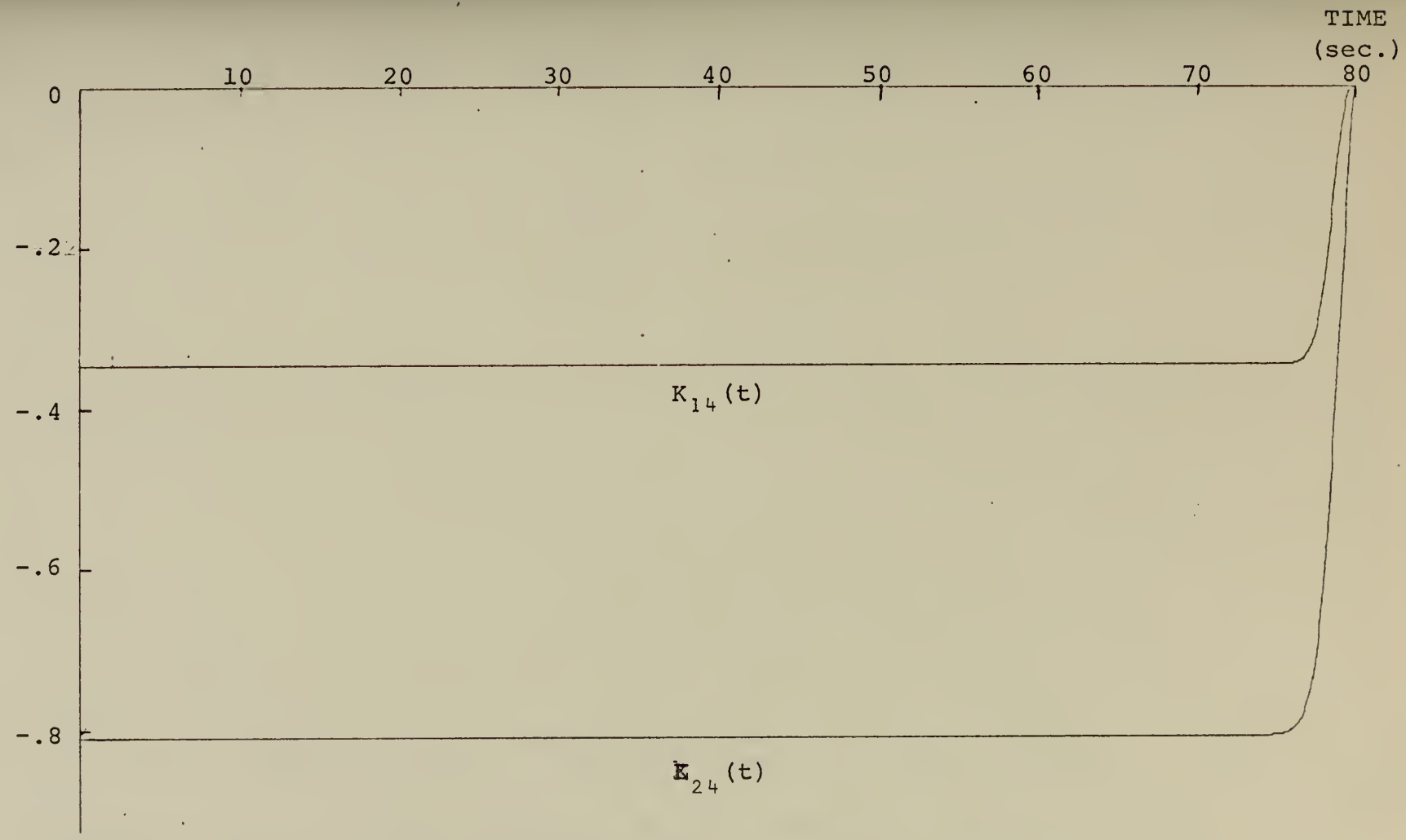

Figure 13. Time Histories of $K_{14}(t)$, $K_{24}(t)$, $K_{41}(t)$ and $K_{42}(t)$. $\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$.

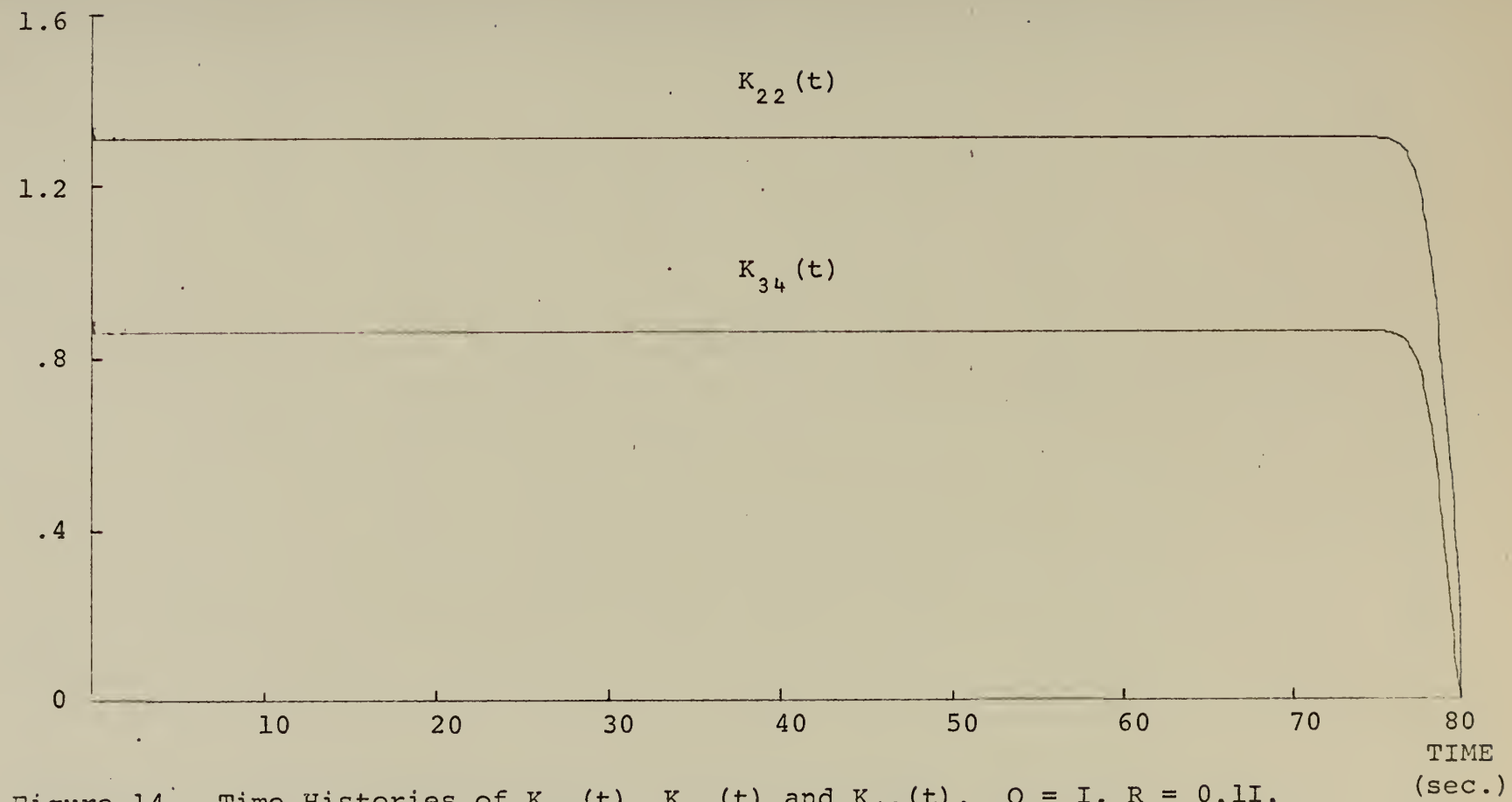

Figure 14. Time Histories of $K_{22}(t)$, $K_{34}(t)$ and $K_{43}(t)$. $\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$.

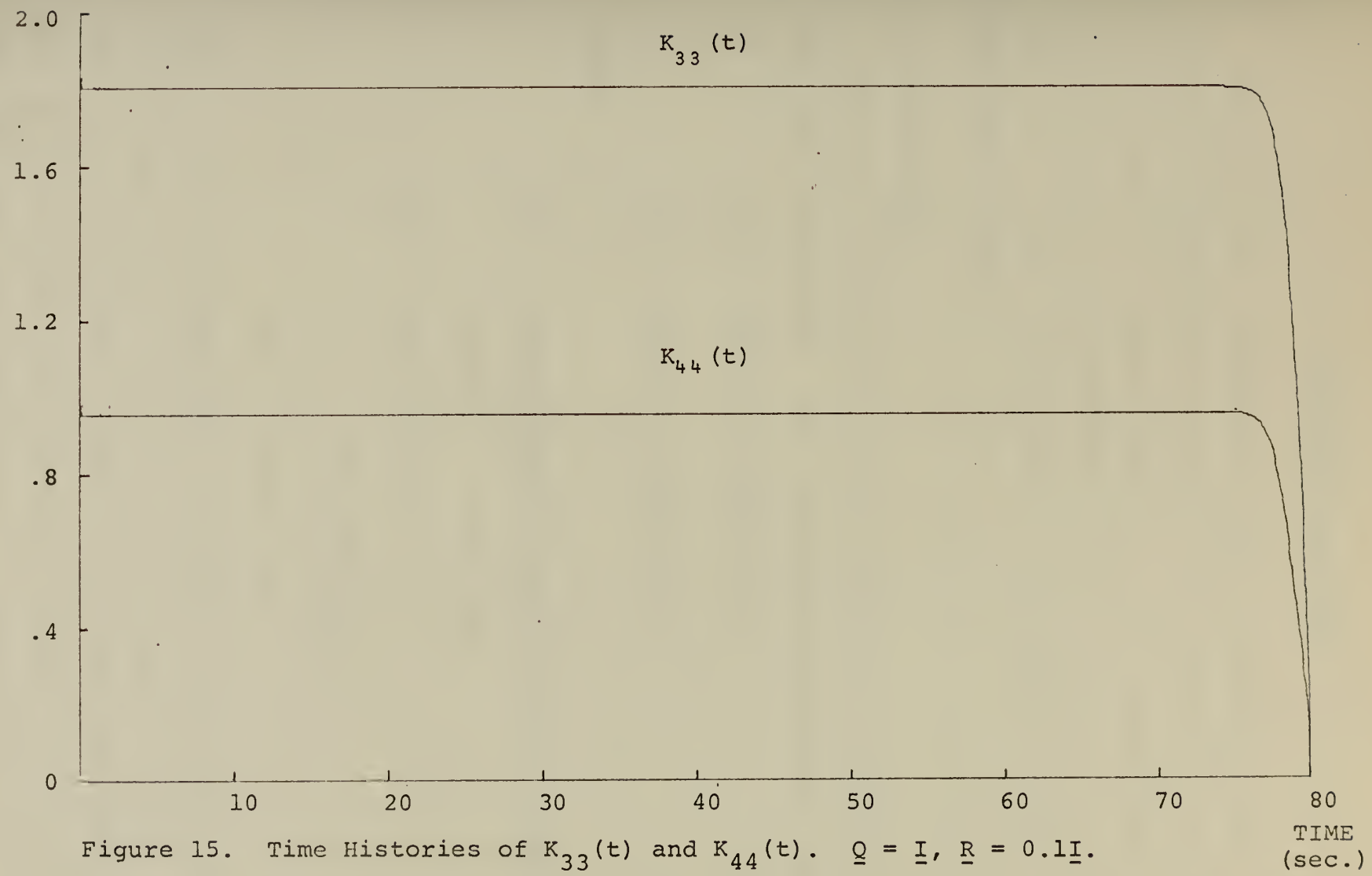

Figure 15. Time Histories of $K_{33}(t)$ and $K_{44}(t)$. $\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$.

V. SIMULATION PROCEDURE

The NPS-360/67 computer was used to solve the ship's time response incorporating the control laws determined previously for various wave forcing functions. The computer program is given following the appendicies, and a flowchart is shown in Fig. 16. In order to solve Eqs. (4.25) and (4.32), $\underline{w}(t)$ and $v(t)$ must be known.

A. WAVE MODEL

The wave forces were estimated using the properties of deep water Airey waves. The resulting equations are [Ref. 3]

$$\hat{z}_w = A\cos(-\hat{\sigma}'\tau)+B\sin(-\hat{\sigma}'\tau) \tag{5.1}$$

$$\hat{M}_w = C\cos(-\hat{\sigma}'\tau)+D\sin(-\hat{\sigma}'\tau) \tag{5.2}$$

where

$$\hat{\sigma}' = \text{non-dimensional wave encounter frequency}$$

$$= (2\pi/\hat{\lambda})^{1/2} - (2\pi\hat{U}/\hat{\lambda})\cos\psi \tag{5.3}$$

$$\psi = \text{wave direction}$$

$$= \begin{matrix} 0° & \text{(following sea)} \\ 180° & \text{(ahead sea)} \end{matrix} \tag{5.4}$$

and $\hat{\lambda}$ is the non-dimensional wavelength of the ocean wave. The coefficients A, B, C and D are graphed versus $\hat{\lambda}$ in Ref. [3]. A least squares polynomial curve fitting subroutine, LSQPL2, was used to fit a sixth-order polynomial to each of the graphs. Each polynomial is a function of $\hat{\lambda}$. The resulting polynomials

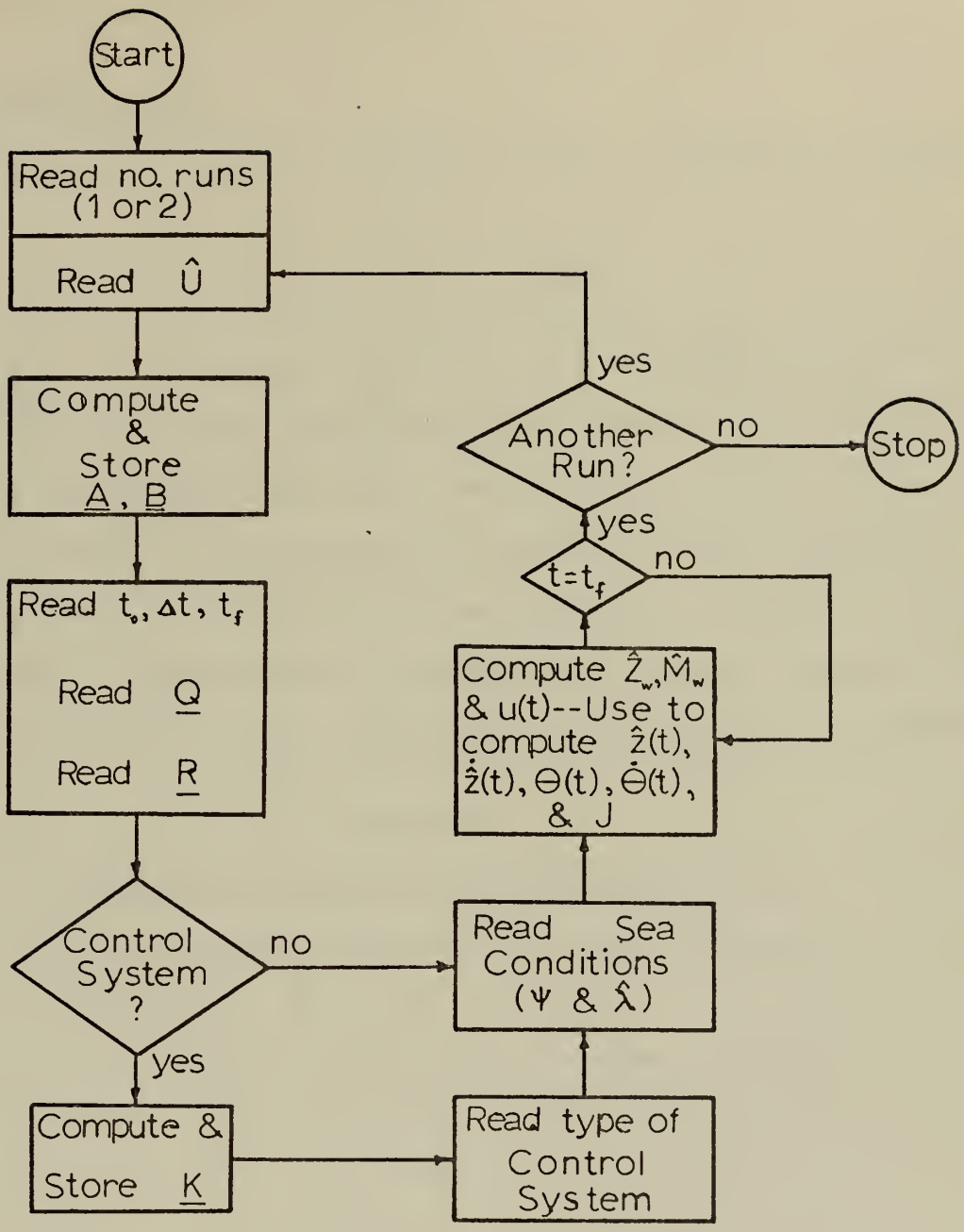

Figure 16. Computer Flowchart.

are used in the computer program to simulate the wave forces and moments acting on the ship.

B. COMMAND SIGNAL

The optimal control law (Eq. (4.36)) requires the command signal

$$\underline{v}(t) = -\underline{R}^{-1}\underline{B}^{T}s(t) \tag{5.5}$$

where $\underline{s}(t)$ is the solution to Eq. (4.25). Equation (5.5) was solved for the three cases shown in Table VI. All three cases are for following seas (ψ=0°) which, as previously stated, is a sea condition which gives an undesirable ship response. The results for case three are shown in Figs. 17 and 18. It is seen that as expected for a sinusoidal forcing function, the command signal is also sinusoidal in form with the exception

TABLE VI

PARAMETERS USED IN SOLVING EQUATION (5.5)

Case No.	$\underline{Q}$	$\underline{R}$	$\hat{U}$	$\hat{\lambda}$	ψ
1	$\underline{I}$	$\underline{I}$	1.0	10.0	0°
2	$\underline{I}$	$0.5\underline{I}$	1.0	10.0	0°
3	$\underline{I}$	$0.1\underline{I}$	1.0	10.0	0°

$\underline{I}$ = Identity Matrix

being where t approaches t_f and the boundary condition

$$\underline{v}(t_f) = \underline{0}$$

must be met.

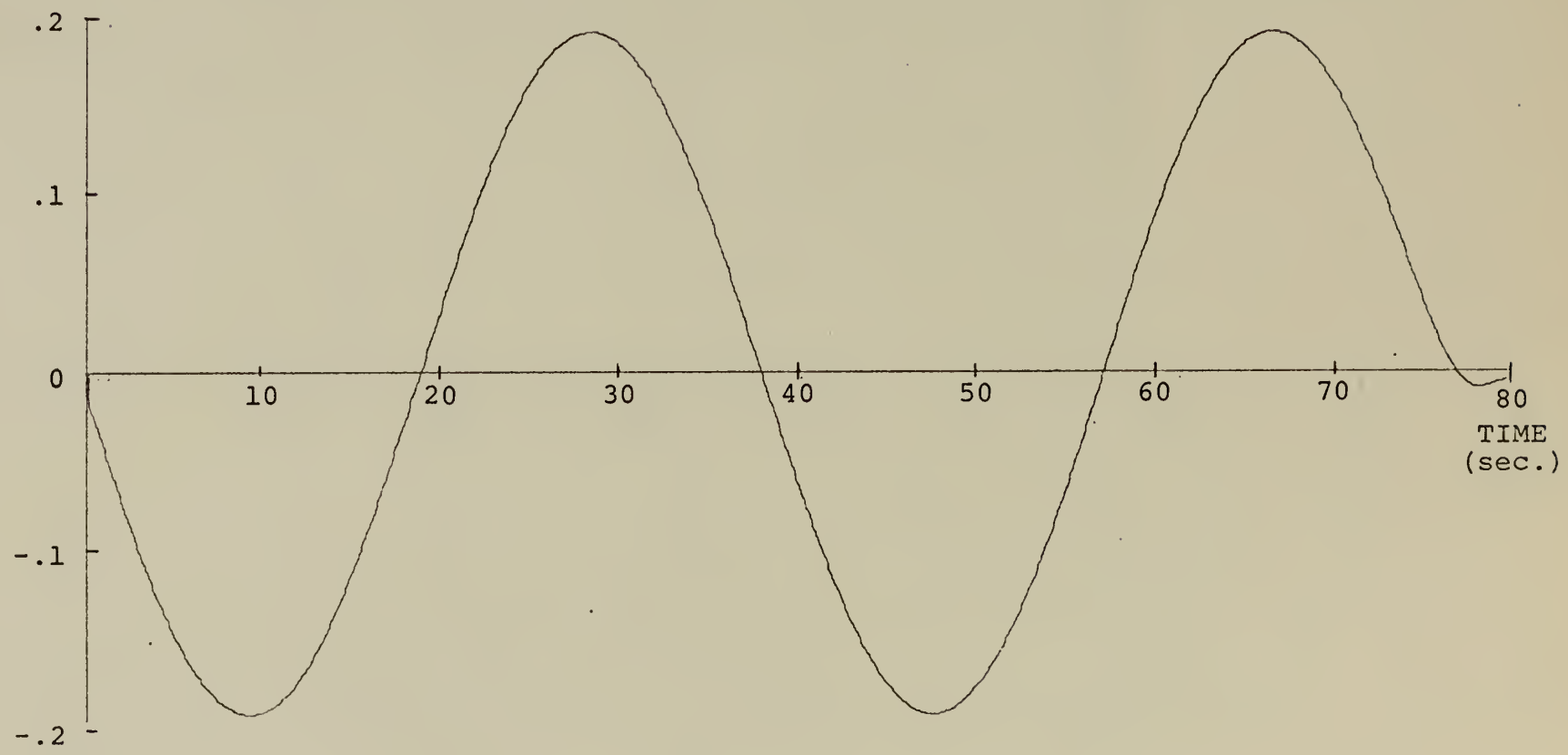

Figure 17. Time History of Command Signal $V_1(t)$.
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.0$, $\psi = 0°$, $\hat{\lambda} = 10$.

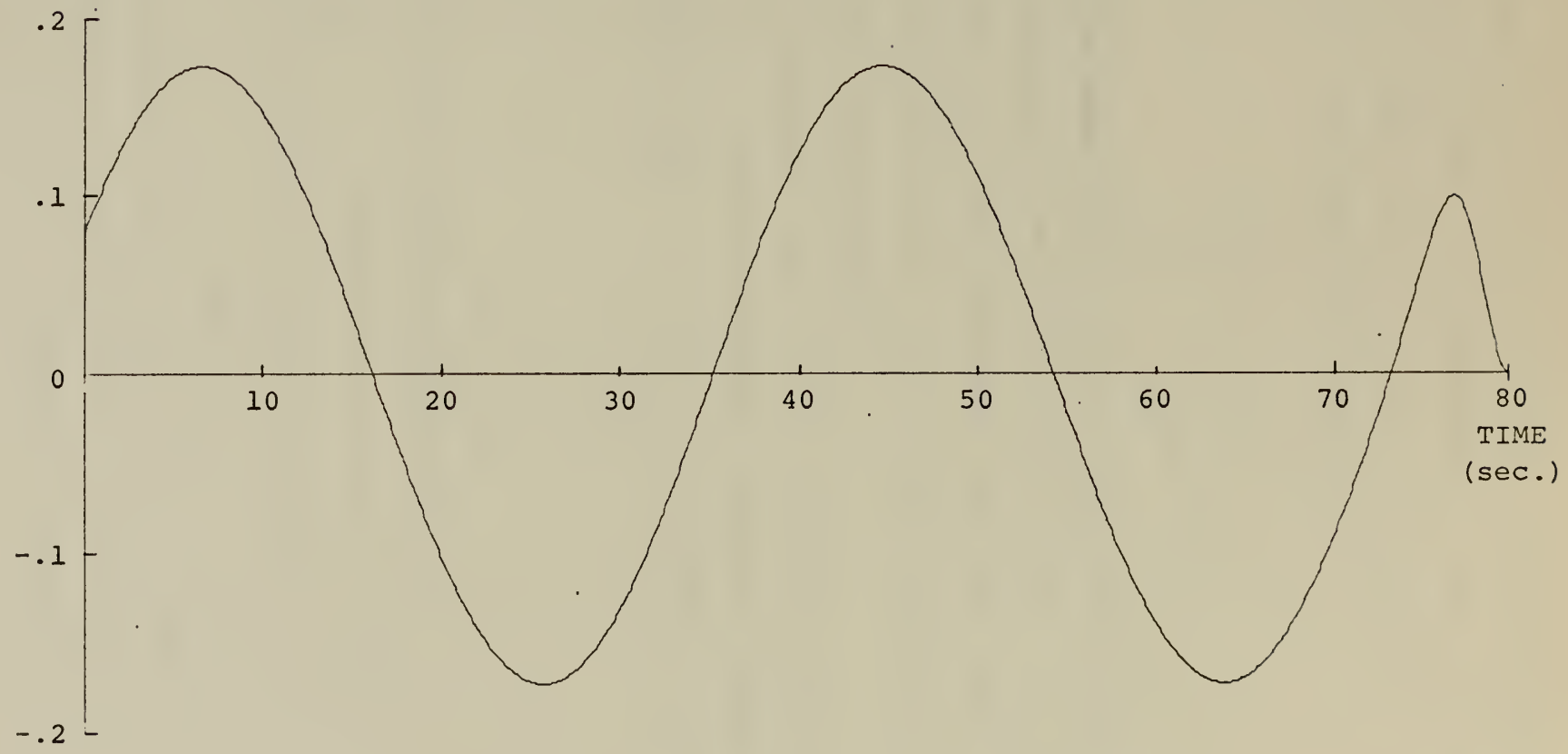

Figure 18. Time History of Command Signal $V_2(t)$.
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.0$, $\psi = 0°$, $\hat{\lambda} = 10$.

Since Eq. (4.25 must be solved forward in time and Eq. (5.5) can only be solved backward in time, it becomes necessary to have a model for $\underline{v}(t)$ to use in solving Eq. (4.25). The model that was chosen was of the form

$$\begin{aligned} v_1(t) &= M_1 \sin(-\hat{\sigma}\tau+\gamma_1) \\ v_2(t) &= M_2 \sin(-\hat{\sigma}\tau+\gamma_2) \end{aligned} \tag{5.6}$$

where the magnitudes, M_1 and M_2, and phase angles, γ_1 and γ_2, were determined from the solutions to Eq. (5.5) and the final time deviations from sinusoidal form were ignored. The models for the three cases in Table VI are incorporated in the computer program as subroutine EST.

An alternative approach for obtaining the command signal is to use a frequency approach instead of integrating Eq. (4.25) backward in time. If a new time variable, τ, is defined as

$$\tau = t_f - t$$

then

$$d\tau = -dt$$

and Eq. (4.25) can be rewritten as

$$\dot{\underline{s}}(\tau) = [A^T - KBR^{-1}B^T]s(\tau) + KCw(\tau) \tag{5.7}$$

which satisfies the initial condition

$$\underline{s}(0) = \underline{0}$$

As previously stated, $[\underline{A}-\underline{B}\underline{R}^{-1}\underline{B}^T\underline{K}]$ is stable and since the transpose of a stable matrix is also stable, Eq. (5.7) is a stable equation in τ (backwards in time t).

The Laplace transform of $s(\tau)$, after invoking the boundary condition, is given by

$$\underline{S}(s) = [s\underline{I}-\underline{A}^T+\underline{K}\underline{B}\underline{R}^{-1}\underline{B}^T]^{-1}\underline{K}\underline{C}\underline{w}(s) \tag{5.8}$$

If $w(\tau)$ is a periodic function with a discrete Fourier spectrum $\underline{W}(f)$, then the spectrum $\underline{S}(f)$ is given by

$$\underline{S}(f) = [j2\pi f\underline{I}-\underline{A}^T+\underline{K}\underline{B}\underline{R}^{-1}\underline{B}^T]^{-1}\underline{K}\underline{C}\underline{W}(f) \tag{5.9}$$

and $\underline{s}(\tau)$ can be obtained by taking the inverse Fourier transform of Eq. (5.9). This approach will hold for any general function $\underline{w}(\tau)$ since the initial condition of Eq. (5.7) is zero.

VI. RESULTS

The three controllers derived in the previous chapter will be called (1) the "constant-gain controller" -- given by Eq. (4.35), (2) the "optimal controller" -- given by Eq. (4.36) and (3) the "suboptimal controller" -- given by Eq. (4.37). Although the constant-gain controller was derived for the calm sea condition, its use was also investigated for the condition where $\underline{w}(t) \neq \underline{0}$. Figures 19 and 20 show the ship's heave and pitch response to a following sea of wavelength 10 when the ship's speed is 1.65 and no control system is used.

A. CASES INVESTIGATED

The following assumptions were made in the first five cases listed below.

(1) The control surface deflections were instantaneous.

(2) The wave forces and moments, $\underline{w}(t)$, were non-zero and exactly known.

(3) The state vector weighting matrix, $\underline{Q}$, was equal to the identity matrix, $\underline{I}$.

(4) Contact of the underwater hulls and control surfaces with the air-water interface was ignored.

(5) Initial conditions which were considered severe

$$\hat{z}(t_0) = -0.5$$

$$\dot{\hat{z}}(t_0) = 0.0$$

$$\theta(t_0) = 0.5 \text{ radians}$$

$$\dot{\theta}(t_0) = 0.0 \text{ radians/sec.}$$

were used (refer to Fig. 2).

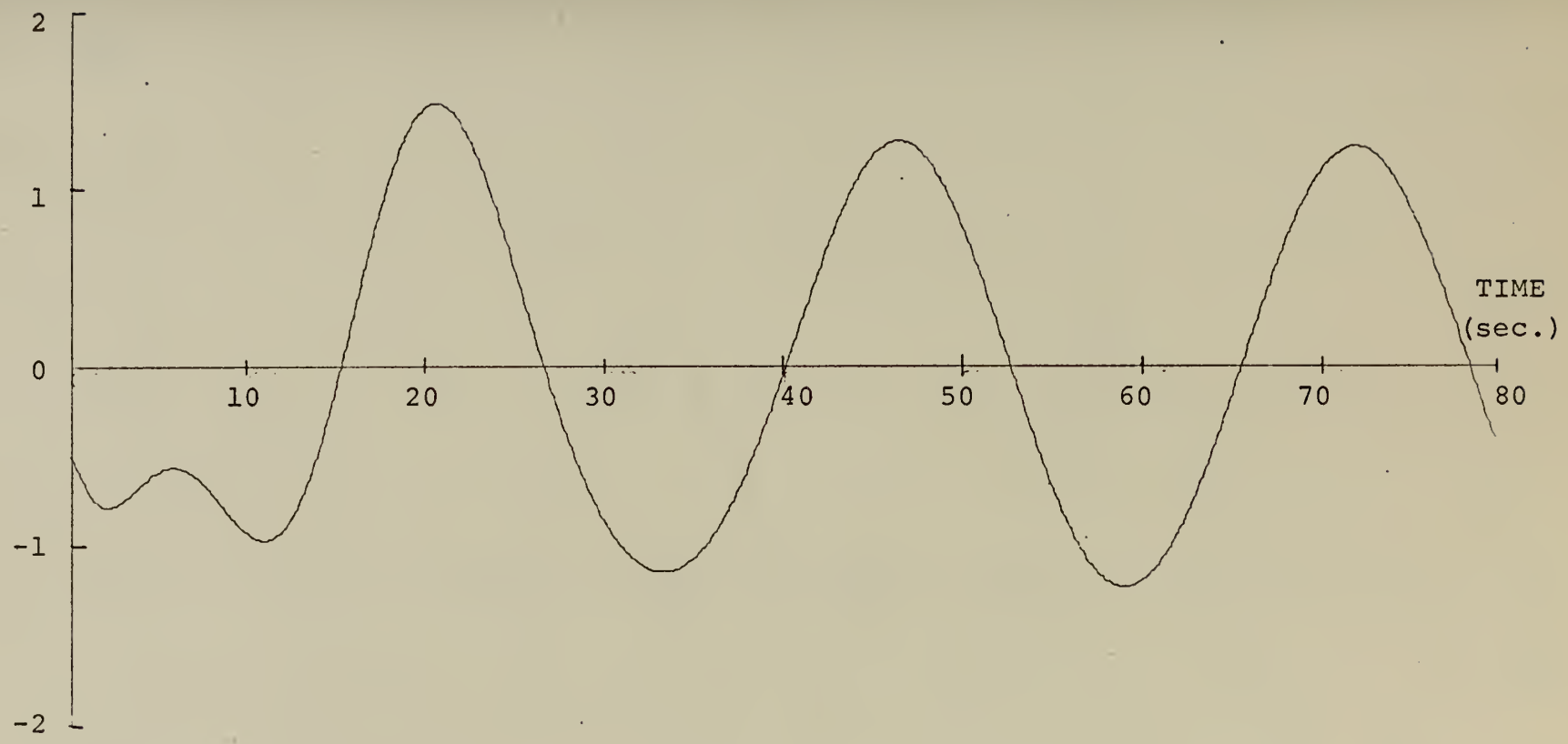

Figure 19. Heave Response with No Control System
$\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$

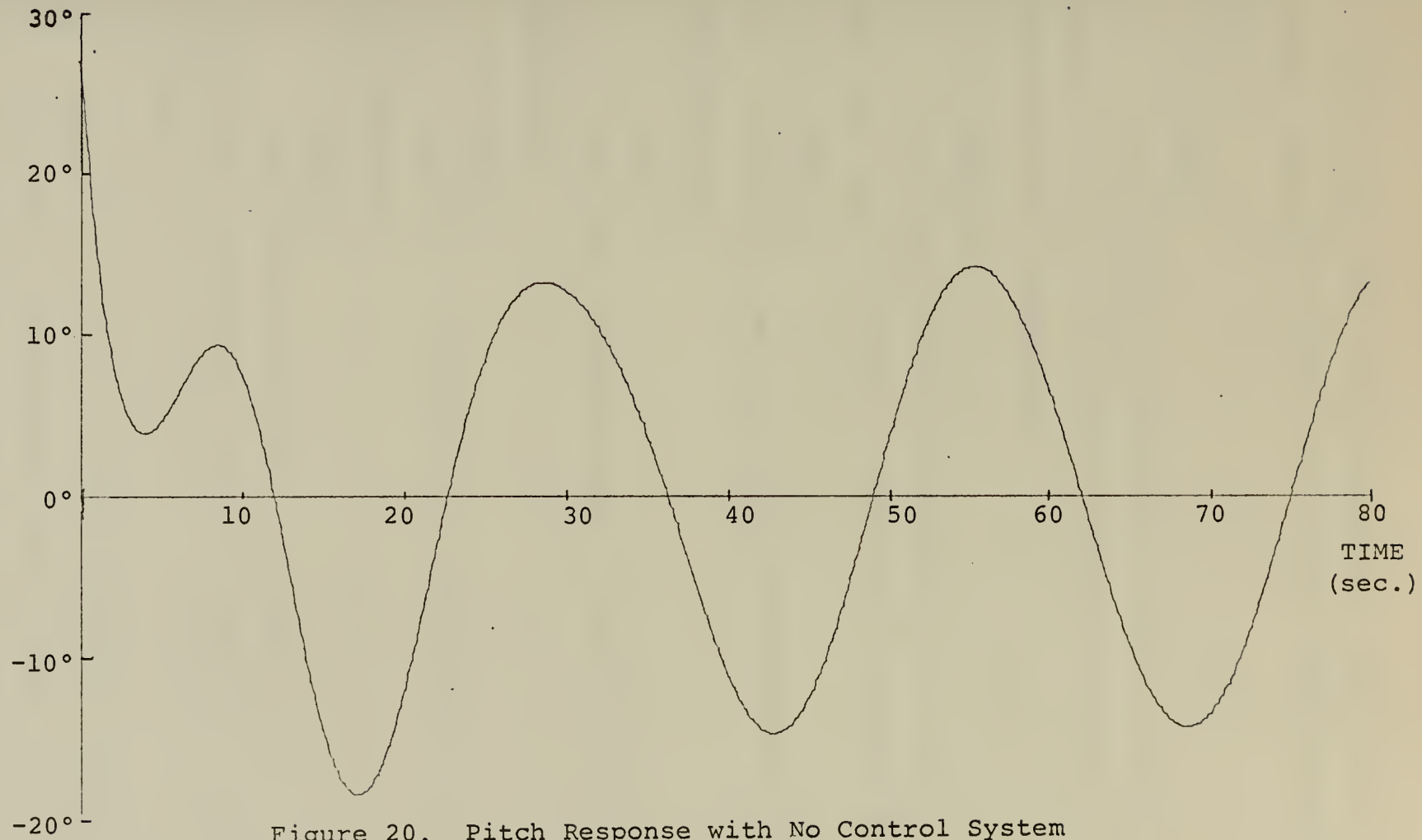

Figure 20. Pitch Response with No Control System
$\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$

In the sixth case it was assumed that $\underline{w}(t)$ was not known exactly; the other assumptions indicated above remained in force.

1. Case I

The reduction of the control weighting matrix $\underline{R}$ and the resultant effect on the ship's response and control surface deflections was investigated.

2. Case II

The control surface deflections were constrained to $\pm 30°$ limits and the effect on the ship's response was determined.

3. Case III

The ship's response using the Constant-Gain Controller was determined.

4. Case IV

The ship's response using the Optimal Controller was determined.

5. Case V

The ship's response using the Suboptimal Controller was determined.

6. Case VI

The effect of imperfect estimates of $\underline{w}(t)$ on the ship's response was investigated using the Suboptimal Controller. The estimate of $\underline{w}(t)$ was perturbed by ± 15 percent of its true value.

B. THE EFFECT OF VARYING CONTROL WEIGHTING MATRIX (CASE I)

The values of the weighting matrix $\underline{R}$ in Eq. (4.1) determine how much emphasis is placed upon the expenditure of control

effort as compared to maintaining the state vector, weighted by $\underline{Q}$, close to the origin. It is seen that as the values of $\underline{R}$ are made smaller compared to those of $\underline{Q}$, more emphasis will be placed upon maintaining $\underline{x}(t)$ close to the origin.

The ship's response in heave and pitch is shown in Figs. 21 and 22 for a following sea with a wavelength of 10 and a ship speed of 1.0. The Optimal Controller was used with two different $\underline{R}$ matrices

$$\underline{R} = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}$$

and

$$\underline{R} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

The figures show, as expected, that heave and pitch are reduced as $\underline{R}$ is reduced. The corresponding increase in control effort is shown in Figs. 23 and 24 for the Elevator and Canard respectively.

C. CONSTRAINED CONTROL SURFACE DEFLECTIONS (CASE II)

A large canard deflection (see Fig. 24) was required to reduce the state vector to its steady-state value when $\underline{R}$ was reduced. The ship's response to the same conditions as in Case I above with $\underline{R} = 0.5\underline{I}$ is shown in Figs. 25 and 26 where the control surface deflections were arbitrarily limited to $\pm$ 30°. The difference in state trajectories between the unconstrained and constrained control surface deflections is almost negligible; additional verification is provided by the

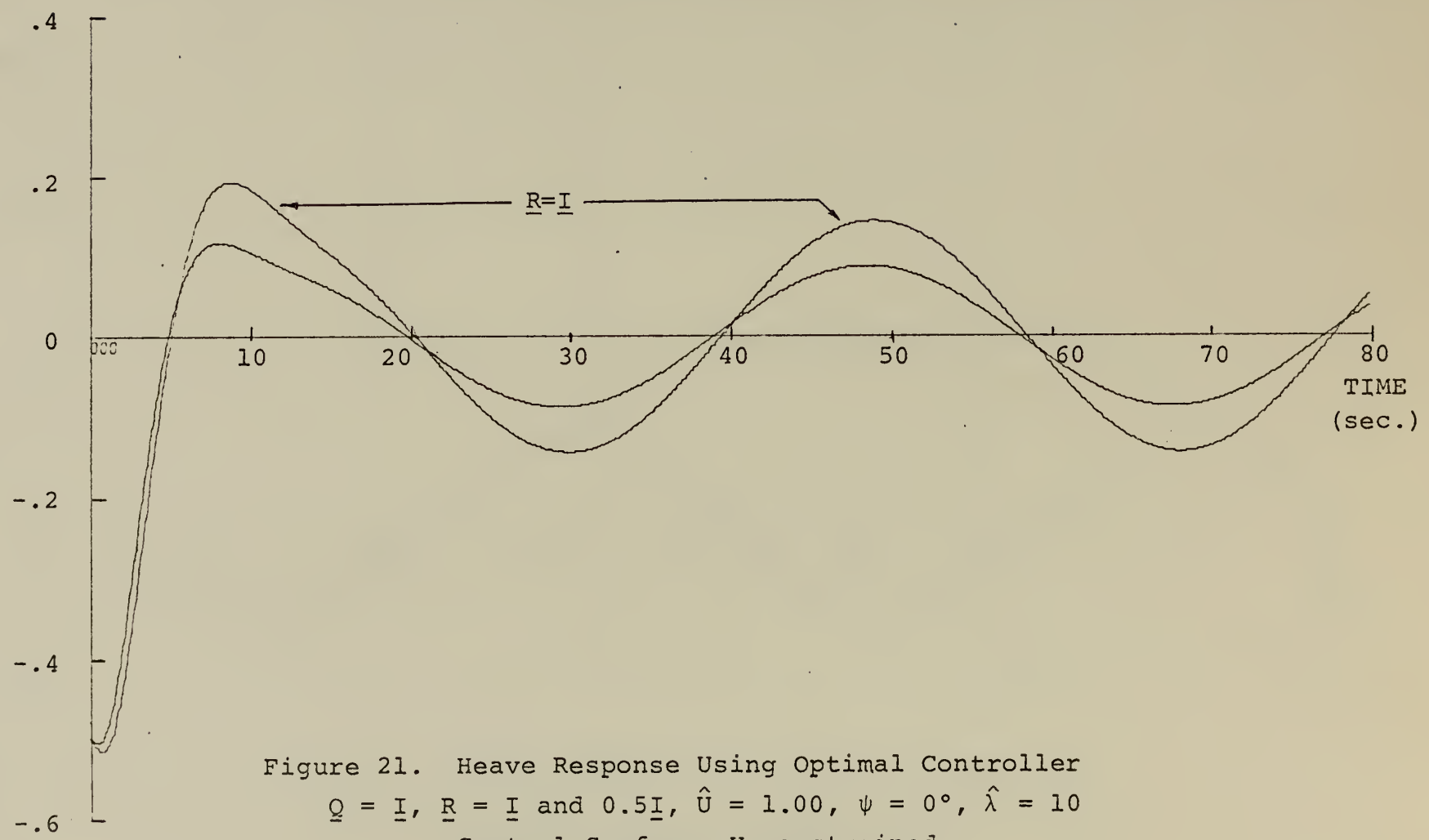

Figure 21. Heave Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = \underline{I}$ and $0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Unconstrained

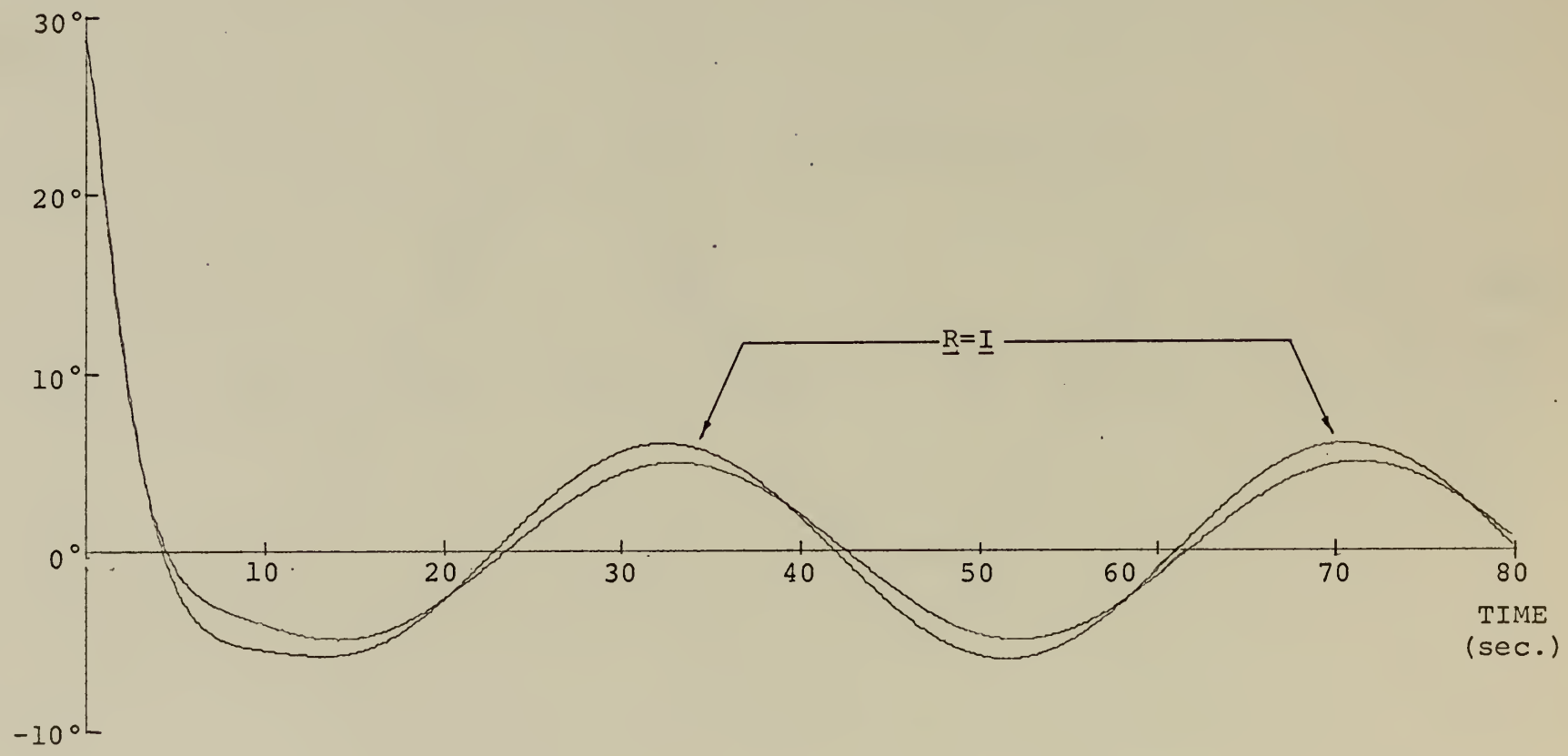

Figure 22. Pitch Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = \underline{I}$ and $0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Unconstrained

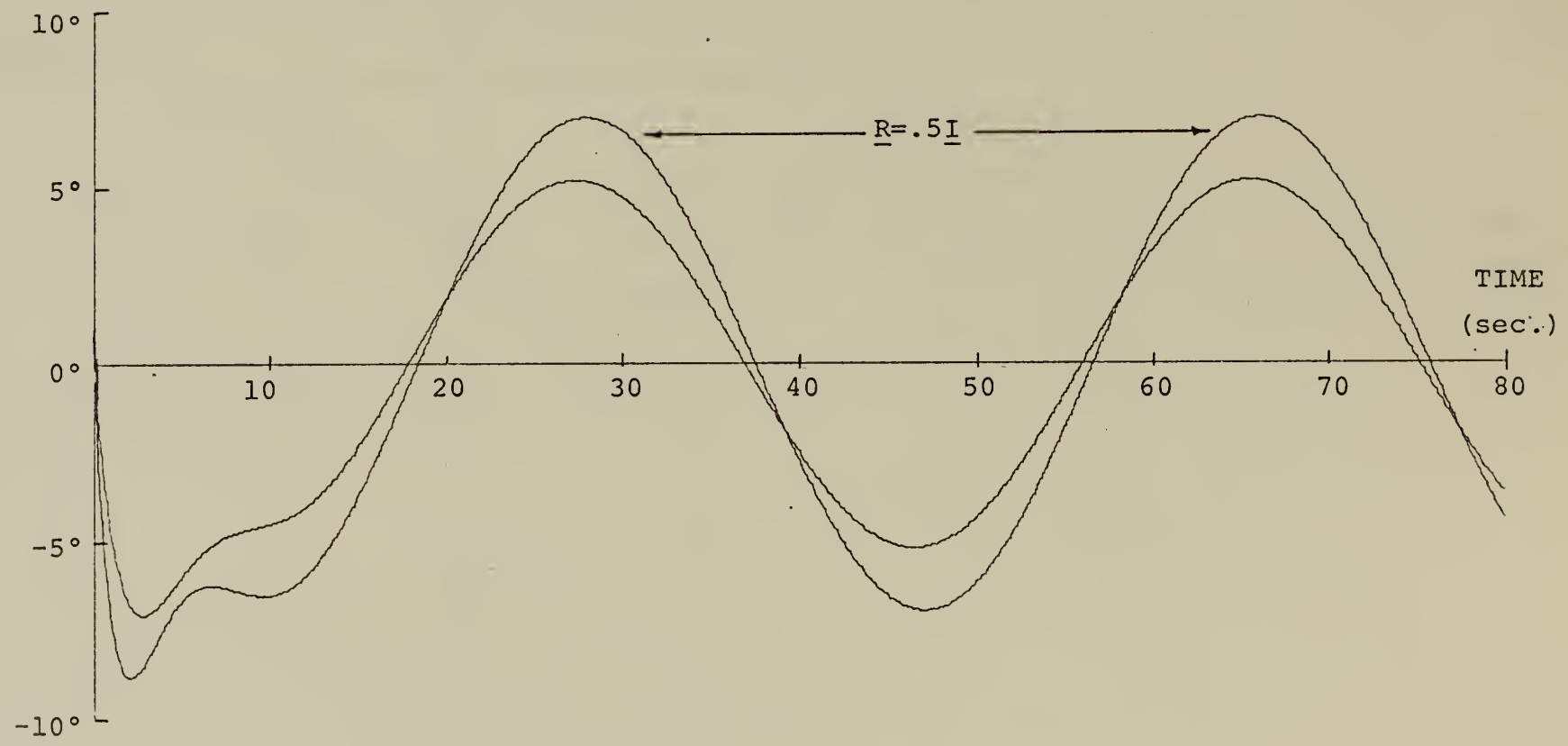

Figure 23. Elevator Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = \underline{I}$ and $0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Unconstrained

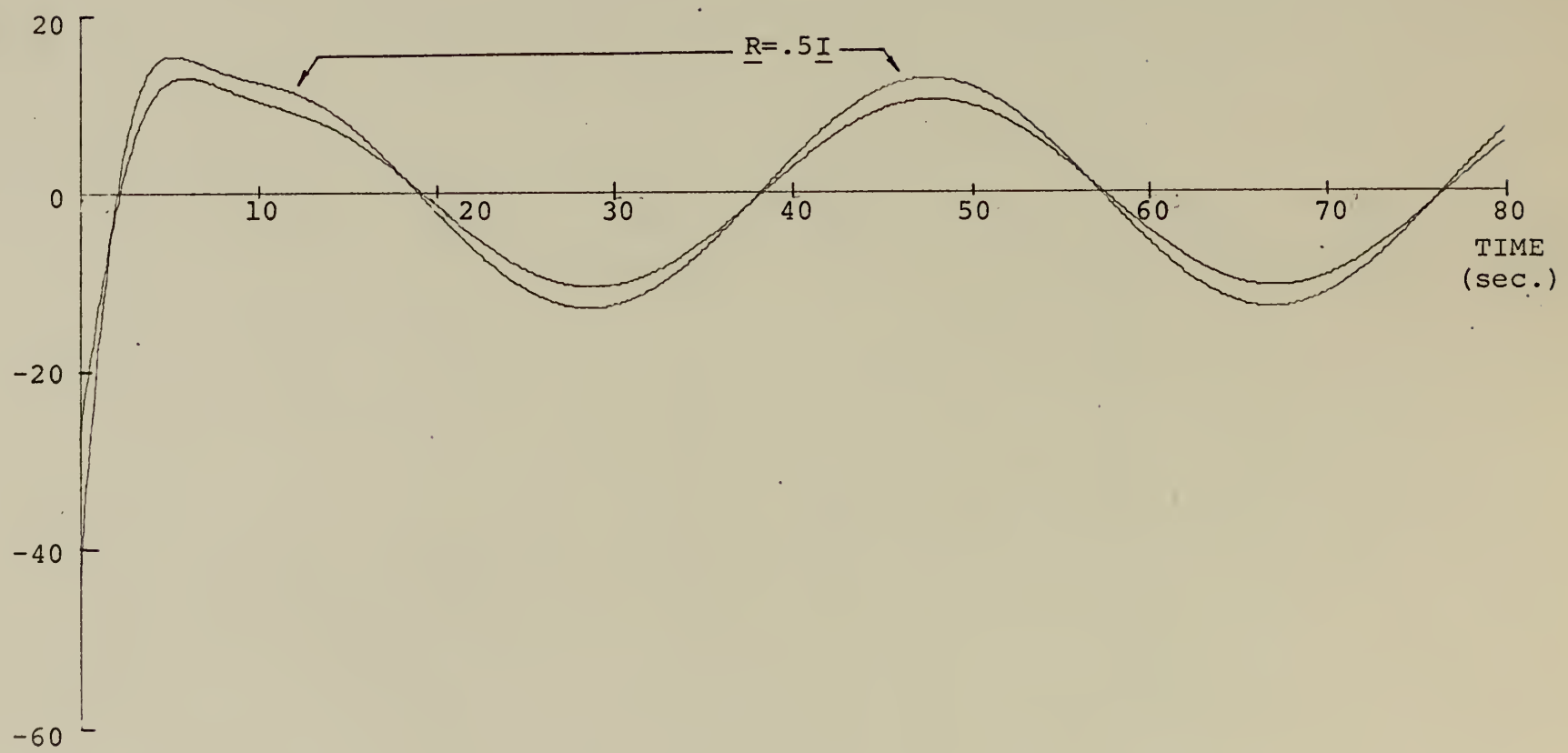

Figure 24. Canard Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = \underline{I}$ and $0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Unconstrained

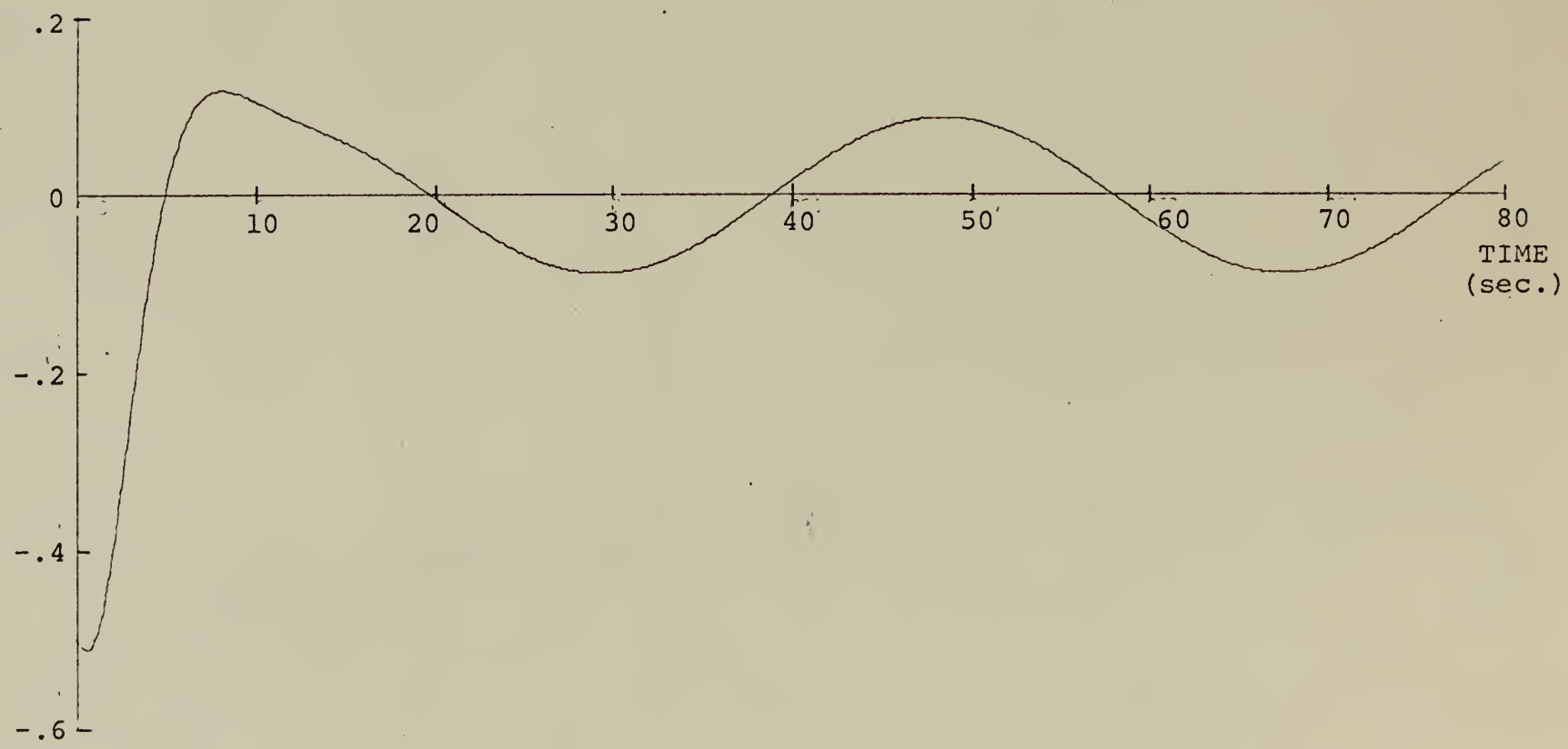

Figure 25. Heave Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

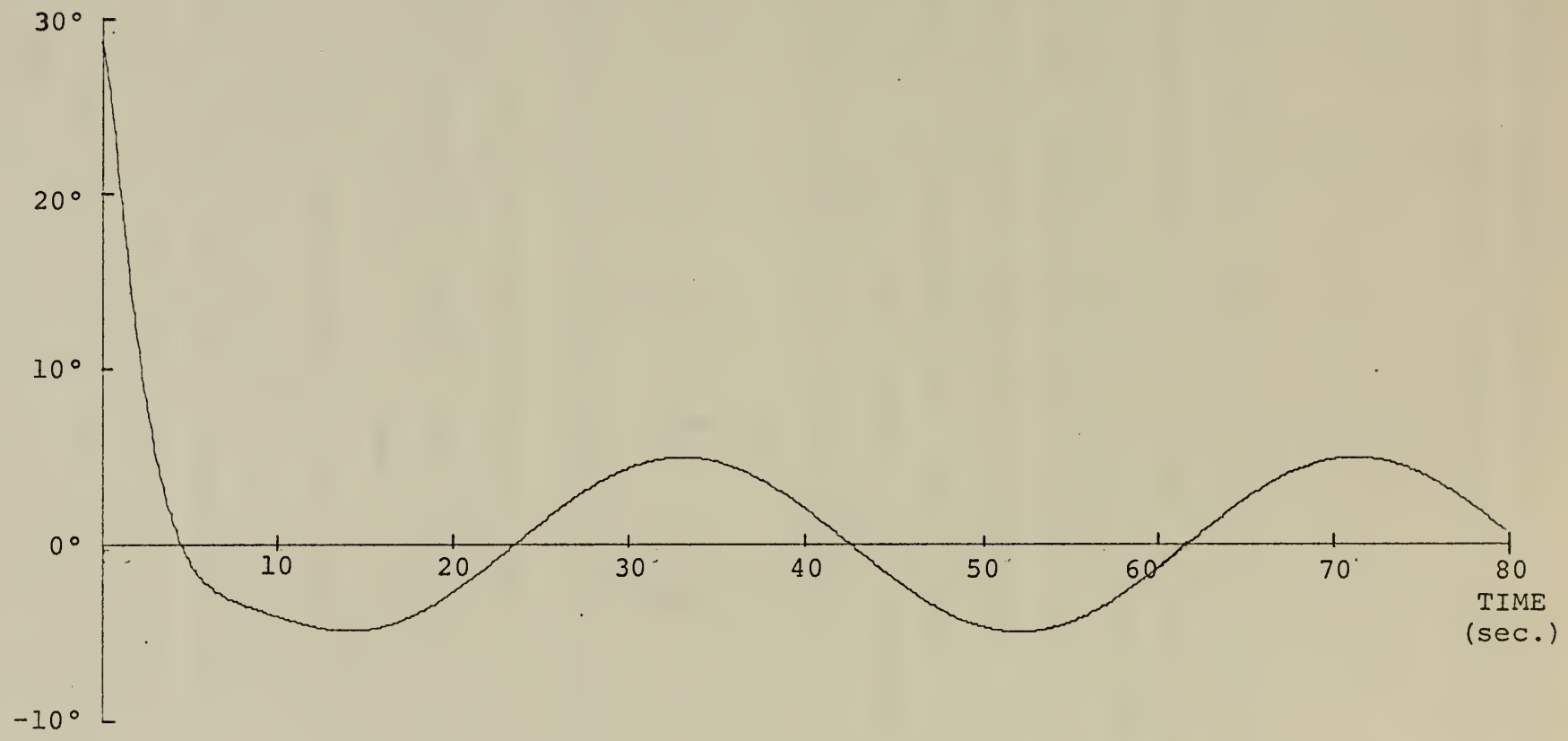

Figure 26. Pitch Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.5\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

performance measure (see Table VII). It was found that an additional reduction in the control weighting matrix to

$$\underline{R} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

brought the state vector much closer to the origin but with unconstrained control deflections, produced unrealistic deflection angles in the initial control period. The control surfaces were then constrained to the previous limits and the resultant differences in the state trajectories were negligible since the period of constraint was small compared to the total control interval $[t_0, t_f]$. As a result, all remaining cases have constrained control surface deflections ($\pm$ 30°) and

$$\underline{R} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

to bring the state vector closer to the origin.

D. THE CONSTANT-GAIN CONTROLLER (CASE III)

As previously stated, this controller was not derived for the case where $\underline{w}(t) \neq 0$. However, since it is based upon feedback of the system states, it will tend to reduce any disturbance of the state vector from its equilibrium point. Figures 27 through 29 show the ship's response and control surface deflections for a following sea with a wavelength of 10 when the ship's speed is 1.65. As expected, the state vector's deviation from equilibrium has been reduced (refer to Figs. 1[illegible] and 20 for comparison) and the Constant-Gain Controller can be used without any knowledge of $\underline{w}(t)$.

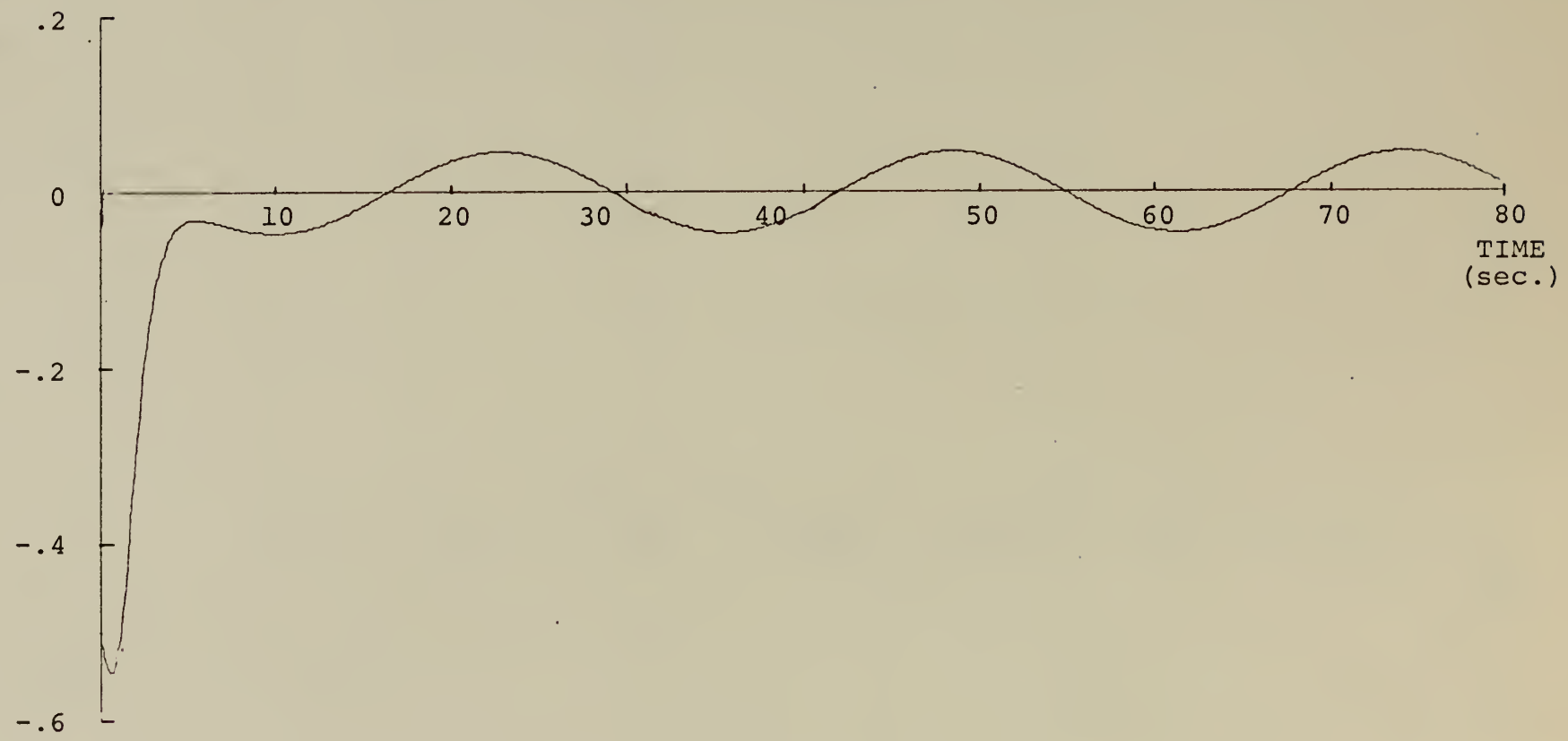

Figure 27. Heave Response Using Constant-Gain Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

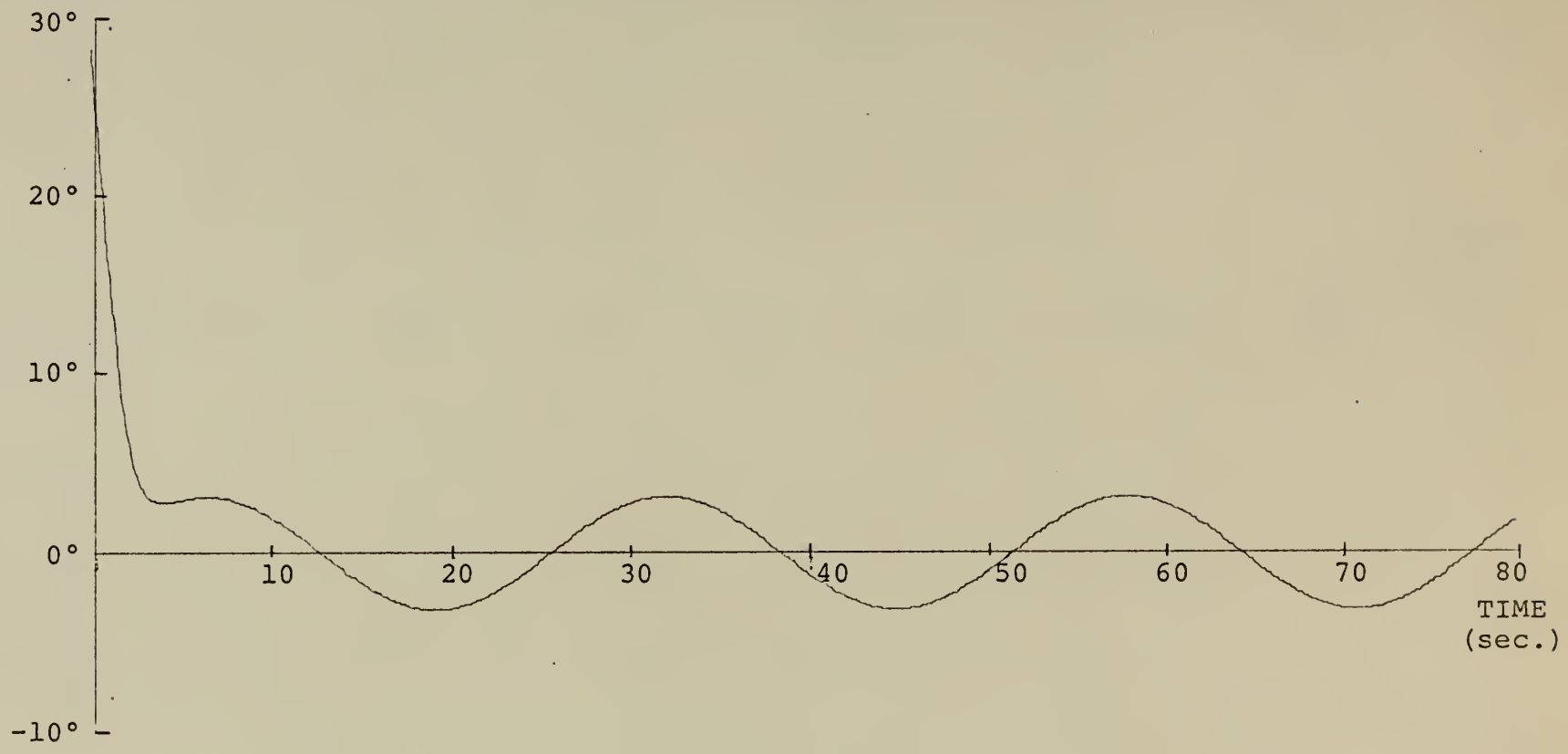

Figure 28. Pitch Response Using Constant-Gain Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

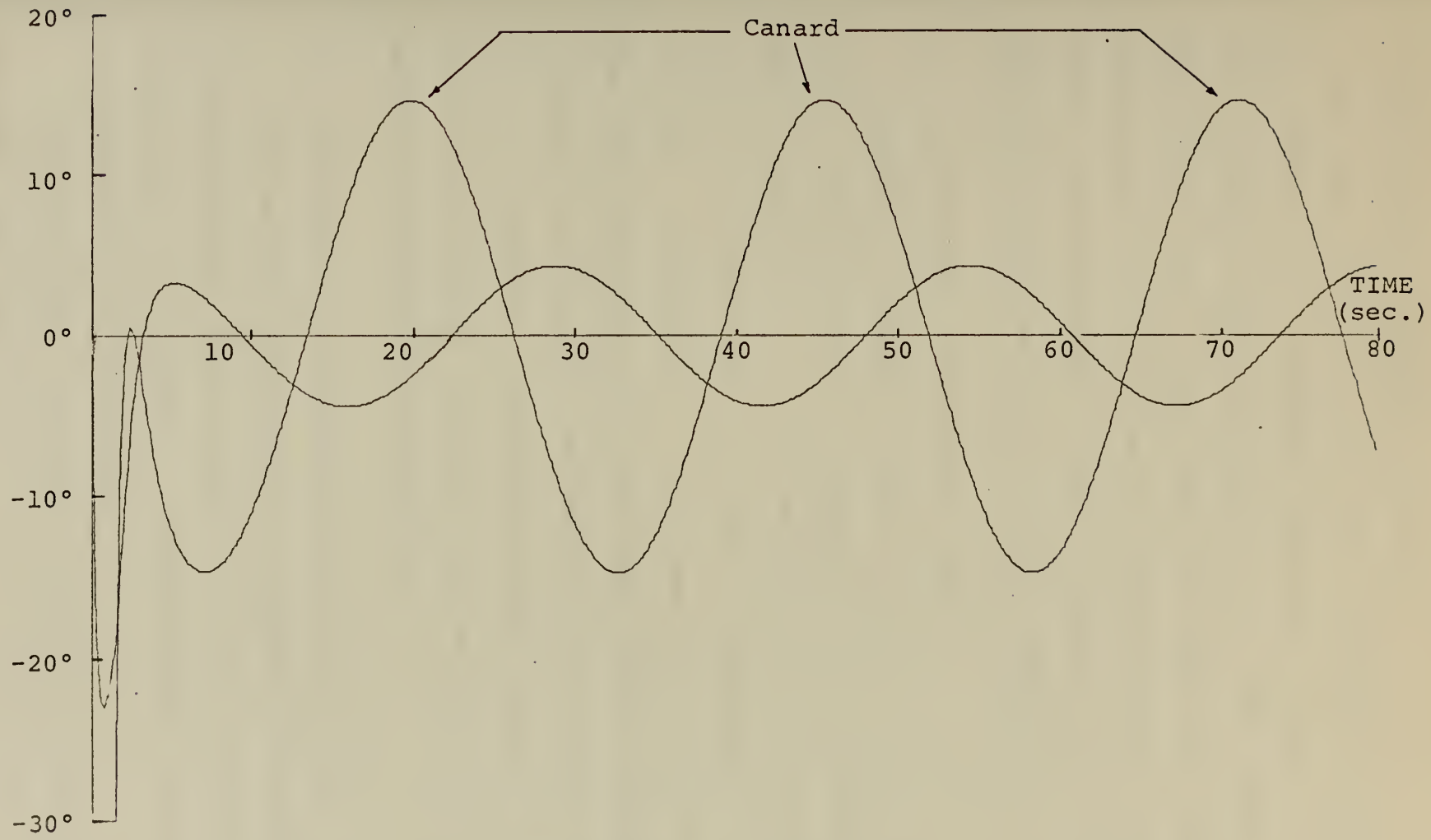

Figure 29. Elevator and Canard Response Using Constant-Gain Controller

$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$

Control Surfaces Constrained to $\pm$ 30°

E. THE OPTIMAL CONTROLLER (CASE IV)

The ship's response using this controller is shown in Figs. 30 and 31. The ship's speed was 1.00 and the seas were following with a wavelength of 10. The control surface deflections (Fig. 32) are seen to be less than those of the Constant-Gain Controller as are the deviations of the state vector from equilibrium. This is expected since the Optimal Controller minimizes the performance measure J.

F. THE SUBOPTIMAL CONTROLLER (CASE V)

Figures 33 through 35 show the ship's response and control surface deflections for a following sea of wavelength 10 and a ship's speed of 1.65. The response and control surface deflections for an ahead sea (ψ = 180°) of wavelength 10 are shown in Figs. 36 through 38. It is seen that, as previously stated, the state approaches zero after the initial transients. Although the states are maintained at the origin after the initial transients, the control effort required is greater than that of the Optimal Controller and, as expected, the performance measure is greater (see Table VII).

G. THE EFFECT OF INACCURATE ESTIMATION OF WAVE FORCE (CASE VI)

The response of the ship to imperfect estimates of $\underline{w}(t)$ was investigated for the case where only the estimates of the amplitudes were imperfect. It was assumed that the wavelength and phase were known exactly.

The Suboptimal Controller was used to control the ship when the amplitude of $\underline{w}(t)$ was not exactly known. The ship was

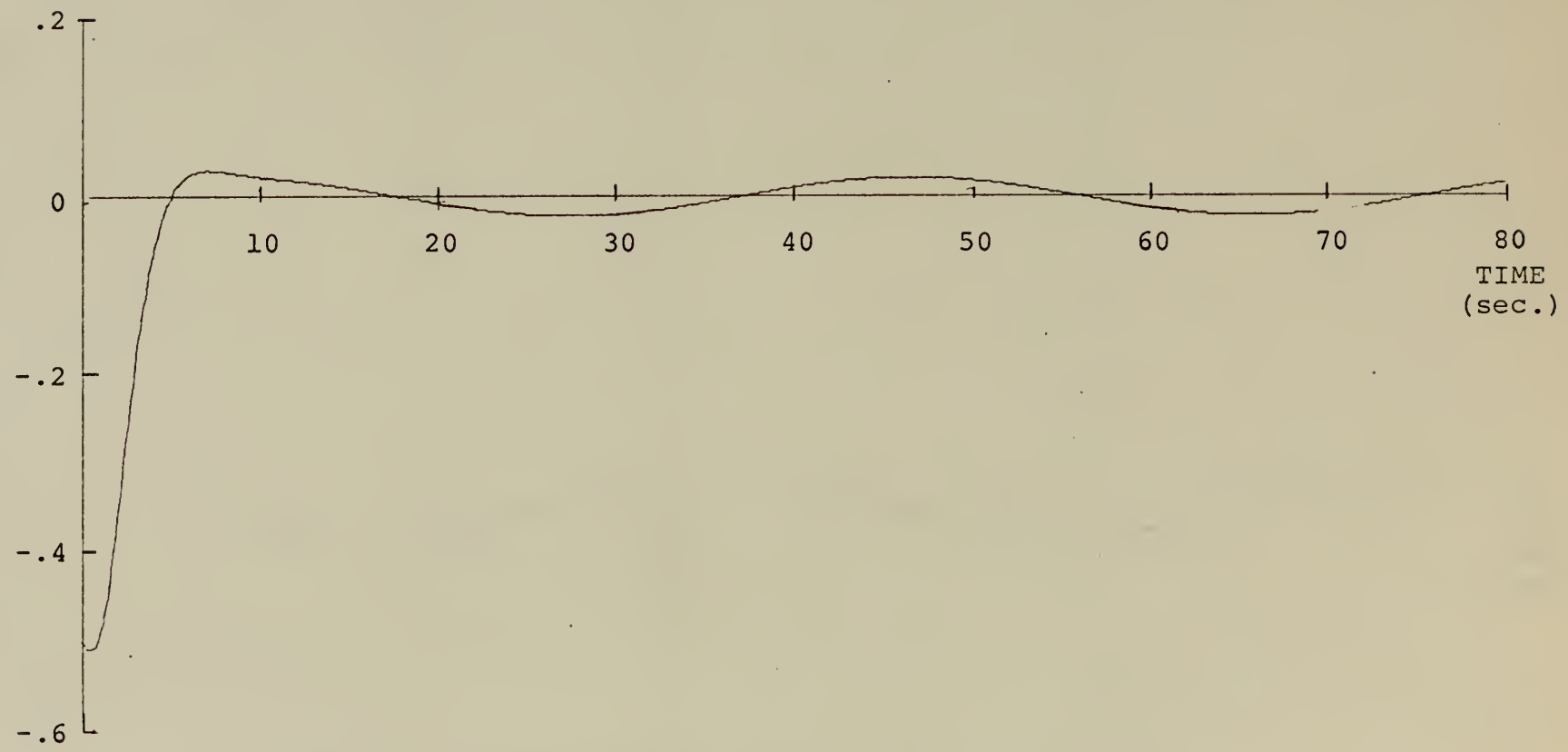

Figure 30. Heave Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

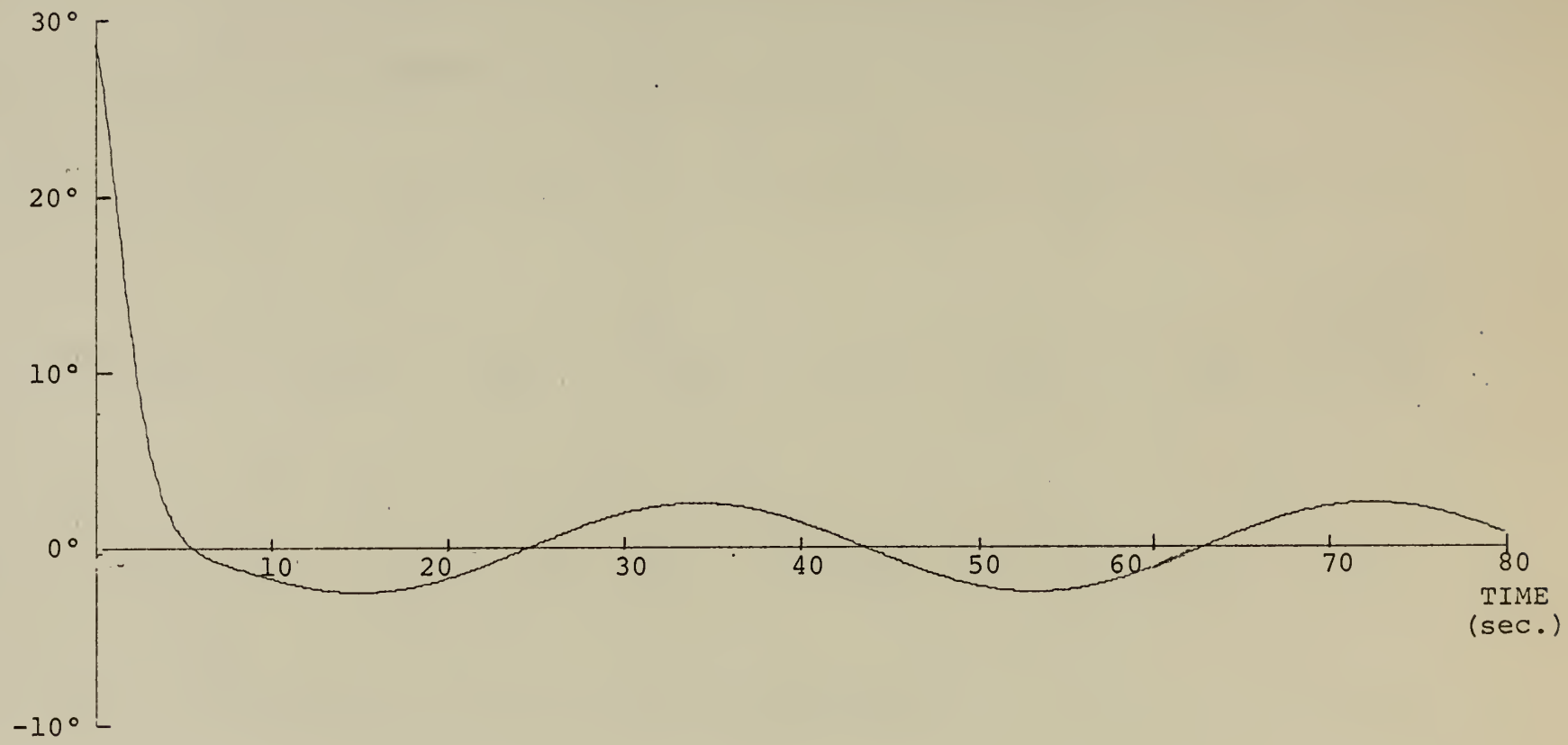

Figure 31. Pitch Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

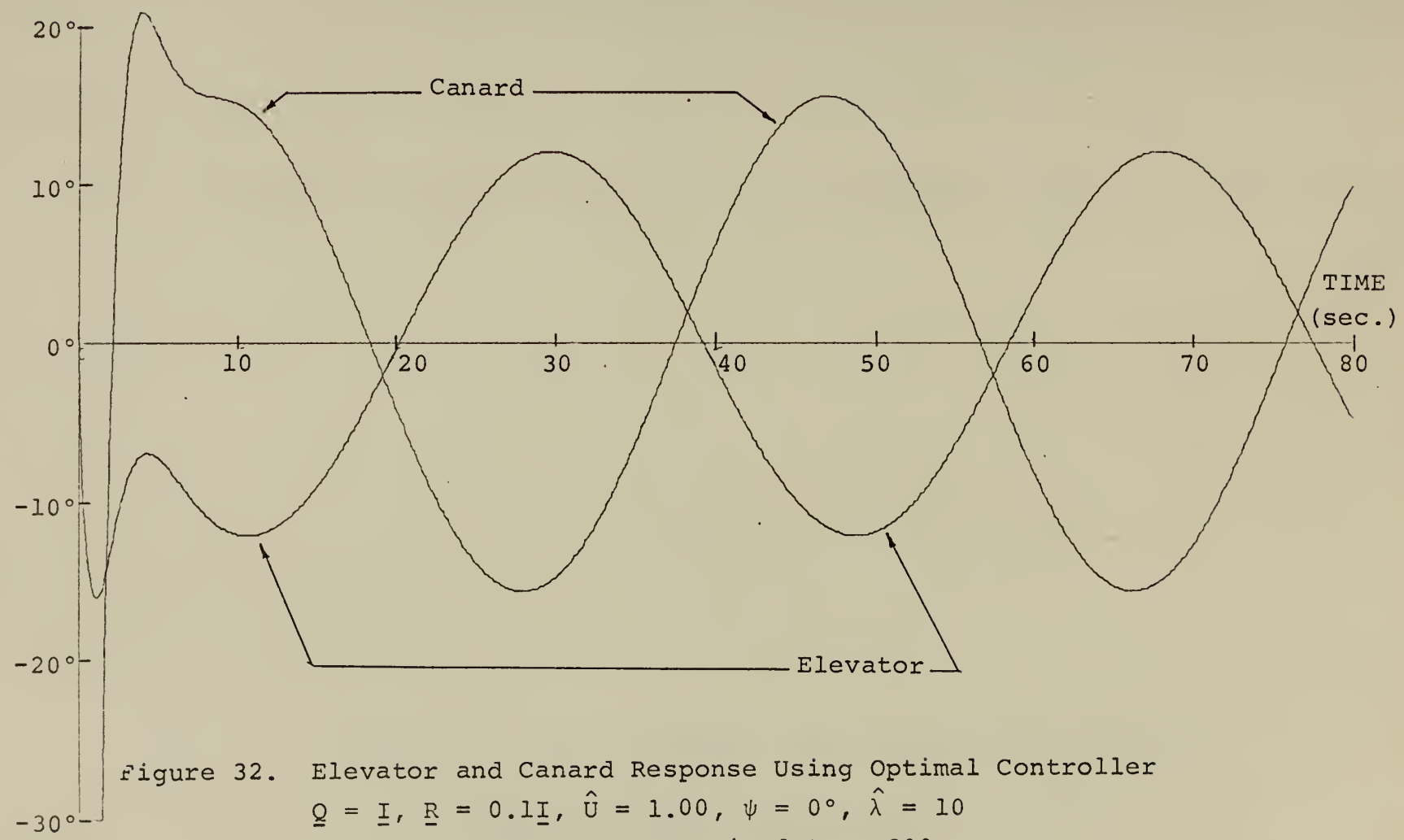

Figure 32. Elevator and Canard Response Using Optimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.00$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

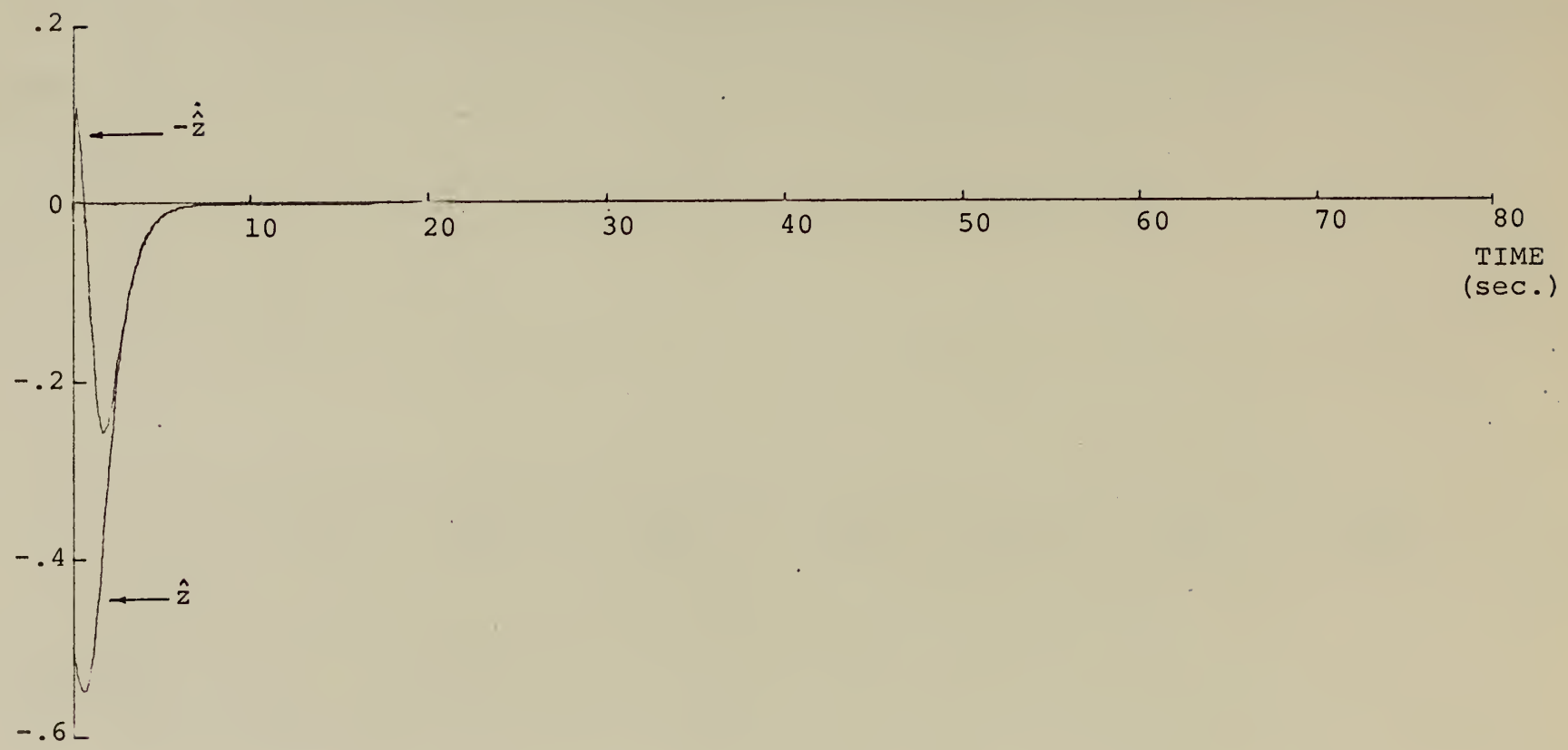

Figure 33. Heave Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $R = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

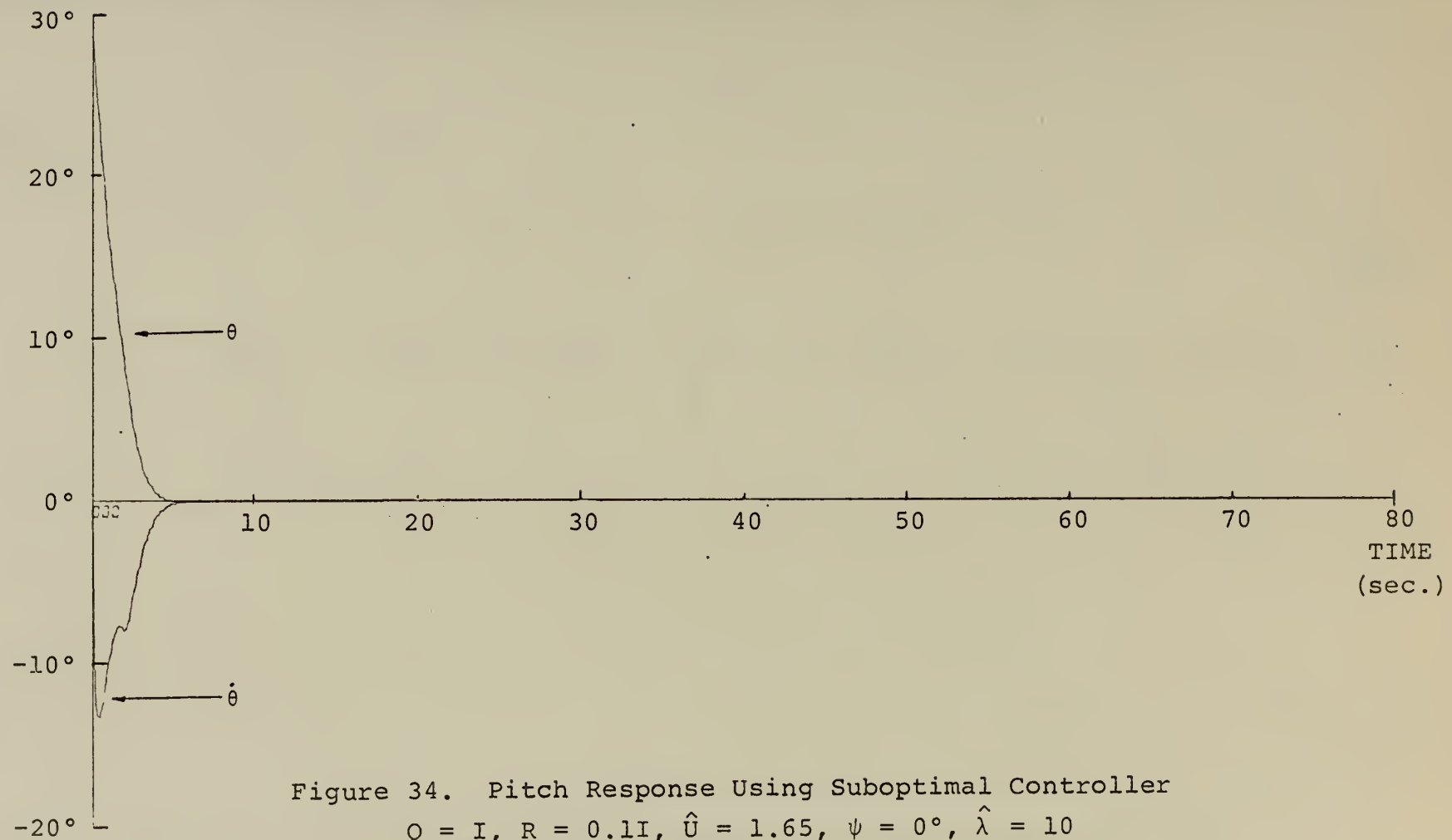

Figure 34. Pitch Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

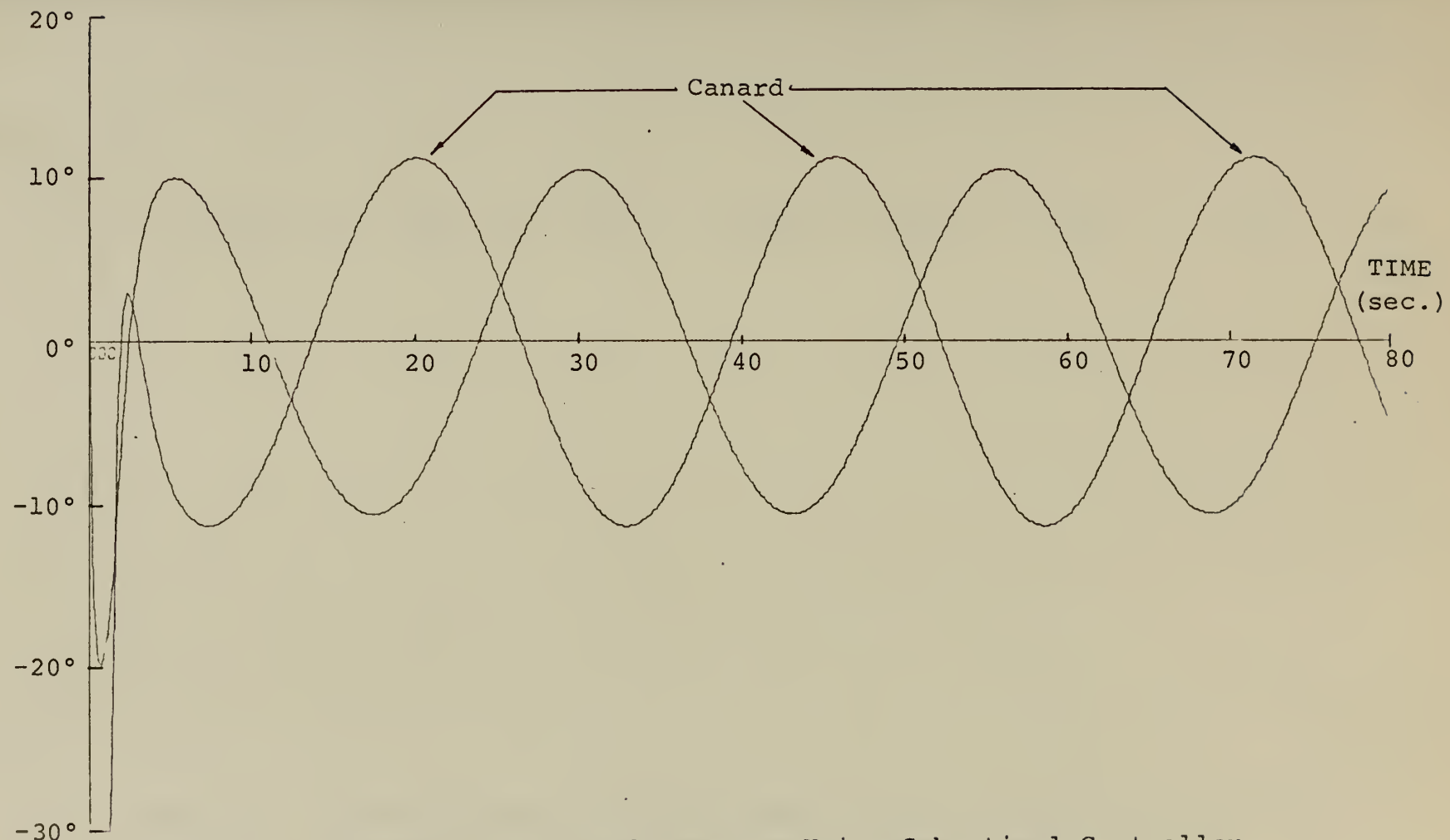

Figure 35. Elevator and Canard Response Using Suboptimal Controller

$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$

Control Surfaces Constrained to $\pm$ 30°

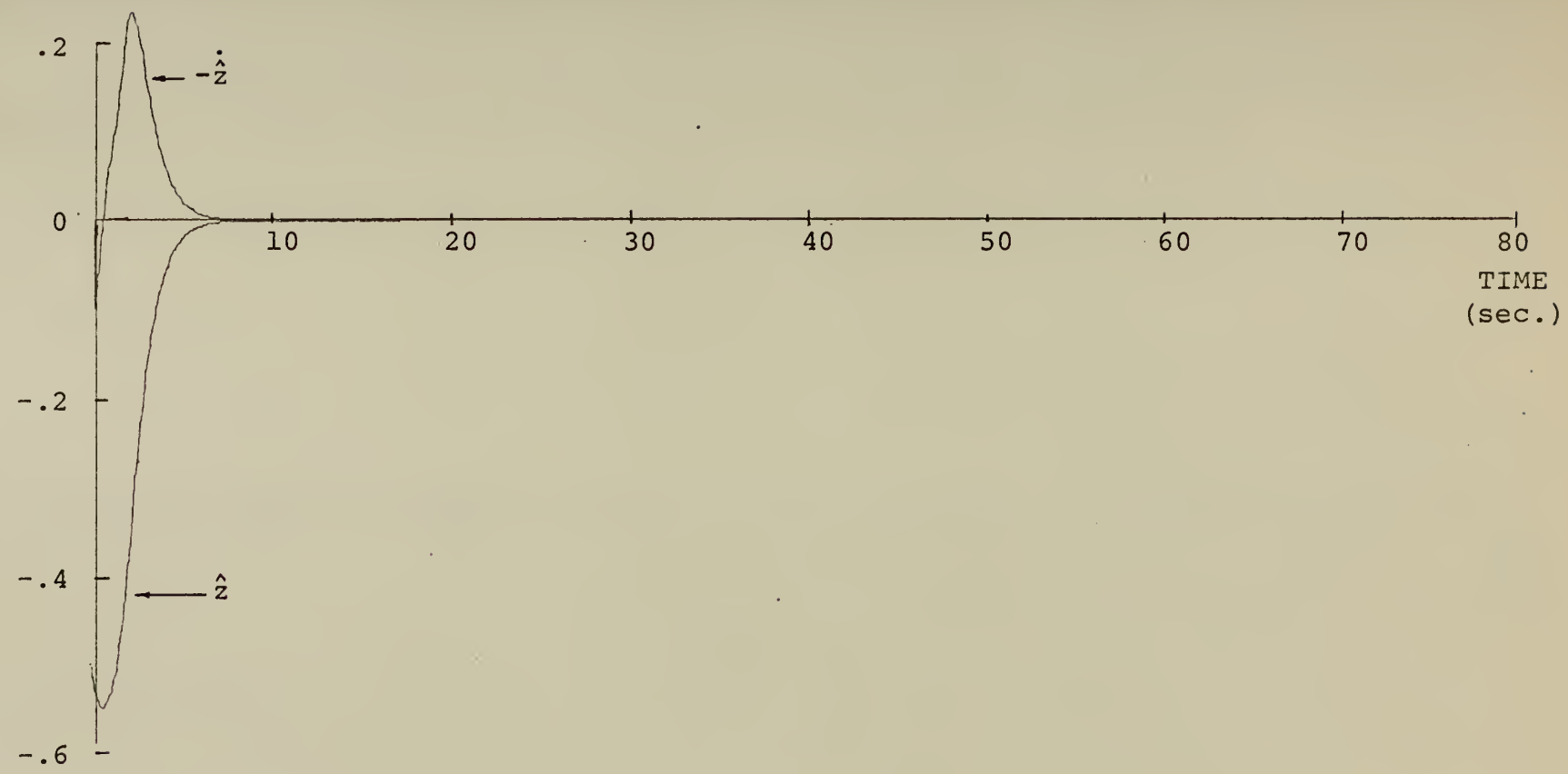

Figure 36. Heave and Heave Velocity Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 180°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

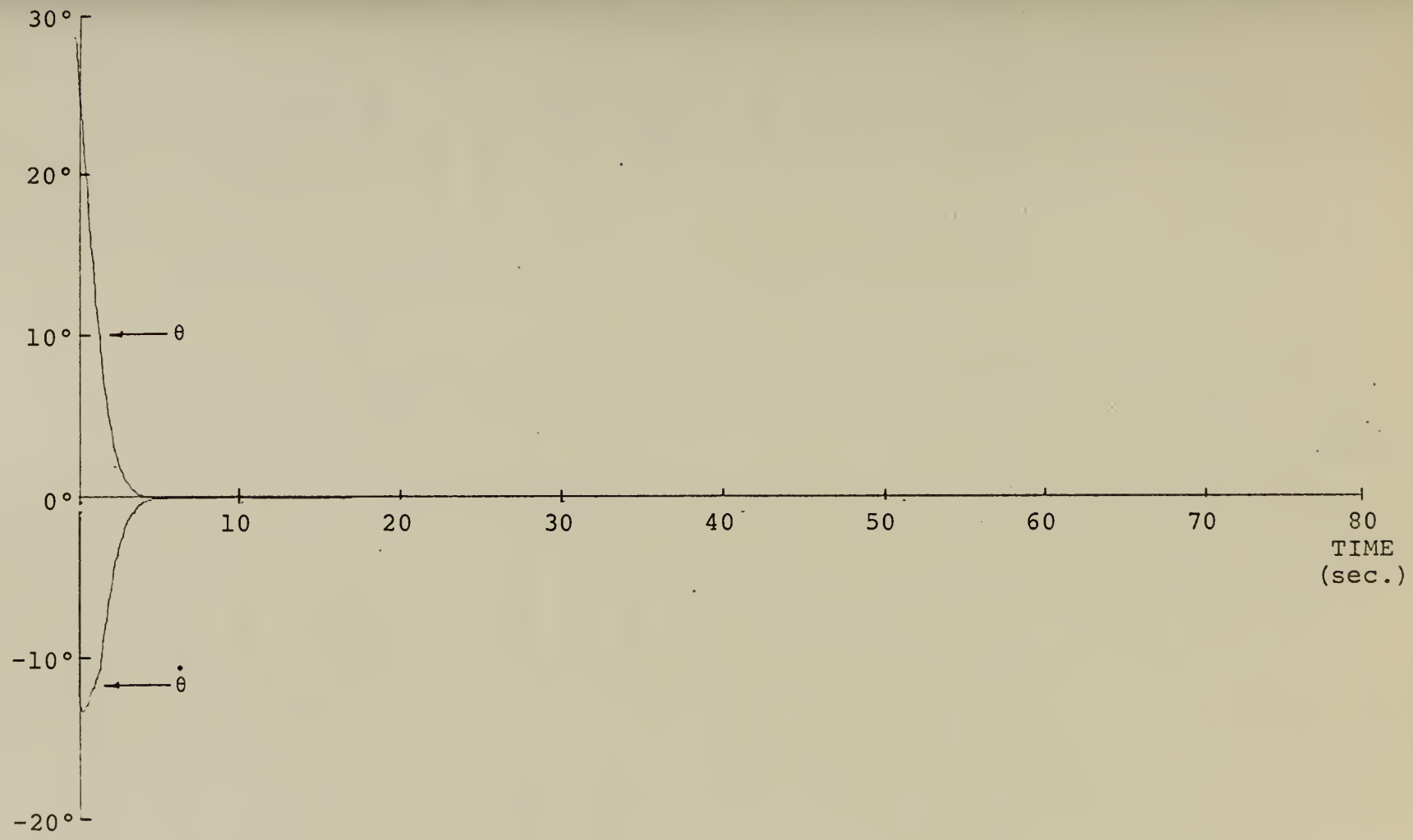

Figure 37. Pitch and Pitch Velocity Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 180°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

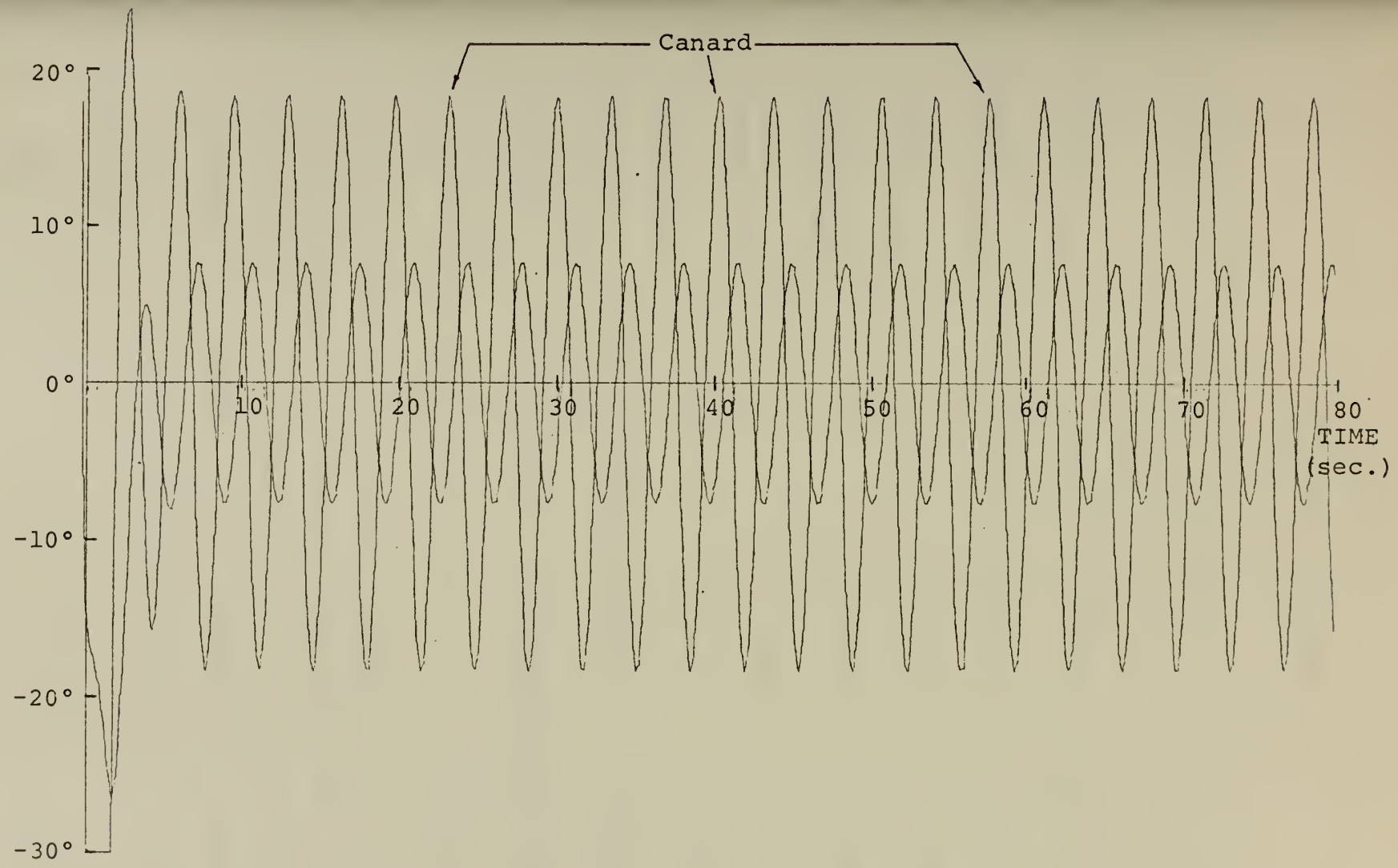

Figure 38. Elevator and Canard Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 180°$, $\hat{\lambda} = 10$
Control Surfaces Constrained to $\pm$ 30°

TABLE VII

PERFORMANCE MEASURES

$t_0 = 0$ sec., $t_f = 80$ sec.

Controller	$\underline{Q}$	$\underline{R}$	$\hat{U}$	ψ	$\hat{\lambda}$	$\underline{w}(t)$	J
None	$\underline{I}$	$0.1\underline{I}$	1.65	0°	10	--	9.328477×10^4
Optimal (unconstrained)	$\underline{I}$	$\underline{I}$	1.00	0°	10	Known	8.5290623×10^2
Optimal (unconstrained)	$\underline{I}$	$0.5\underline{I}$	1.00	0°	10	Known	6.5330823×10^2
Optimal (constrained)	$\underline{I}$	$0.5\underline{I}$	1.00	0°	10	Known	6.5468393×10^2
Constant-Gain (constrained)	$\underline{I}$	$0.1\underline{I}$	1.65	0°	10	Known	3.5693990×10^2
Optimal (constrained)	$\underline{I}$	$0.1\underline{I}$	1.00	0°	10	Known	3.9204965×10^2
Suboptimal (constrained)	$\underline{I}$	$0.1\underline{I}$	1.65	0°	10	Known	3.2483721×10^2
Suboptimal (constrained)	$\underline{I}$	$0.1\underline{I}$	1.65	180°	10	Known	3.9344051×10^2
Suboptimal (constrained)	$\underline{I}$	$0.1\underline{I}$	1.65	0°	10	Underestimated by 15 percent	3.3556745×10^2

running in a following sea of wavelength 10 at a speed of 1.65. The ship's response in heave and pitch are shown in Figs. 39 and 40 where the amplitude of $\underline{w}(t)$ was overestimated and underestimated by 15 percent. It is seen that the estimate error has very little effect on the performance of the Suboptimal Controller.

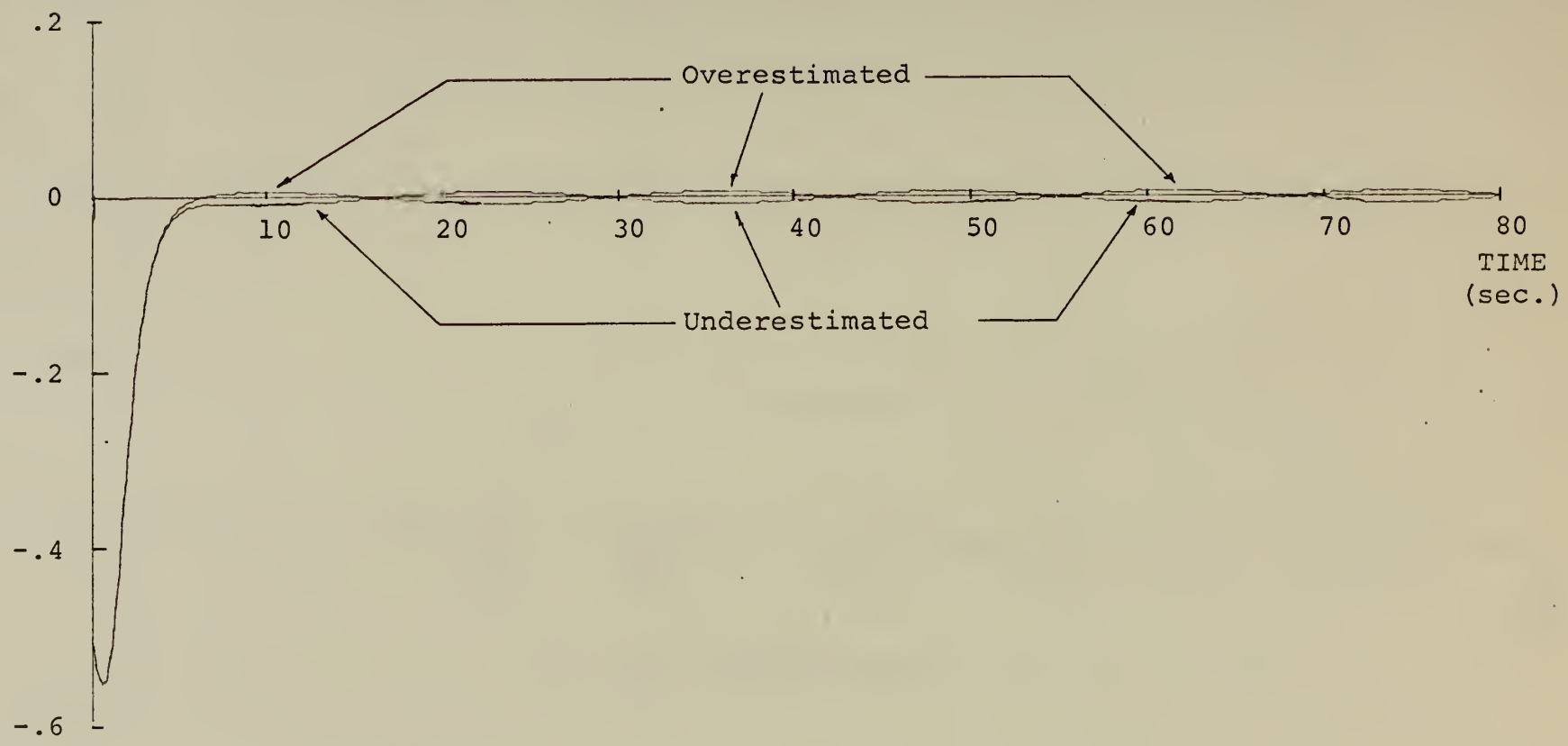

Figure 39. Heave Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$, $\underline{w}(t)$ overestimated
and underestimated by 15 percent, Control Surfaces Constrained to $\pm$ 30°

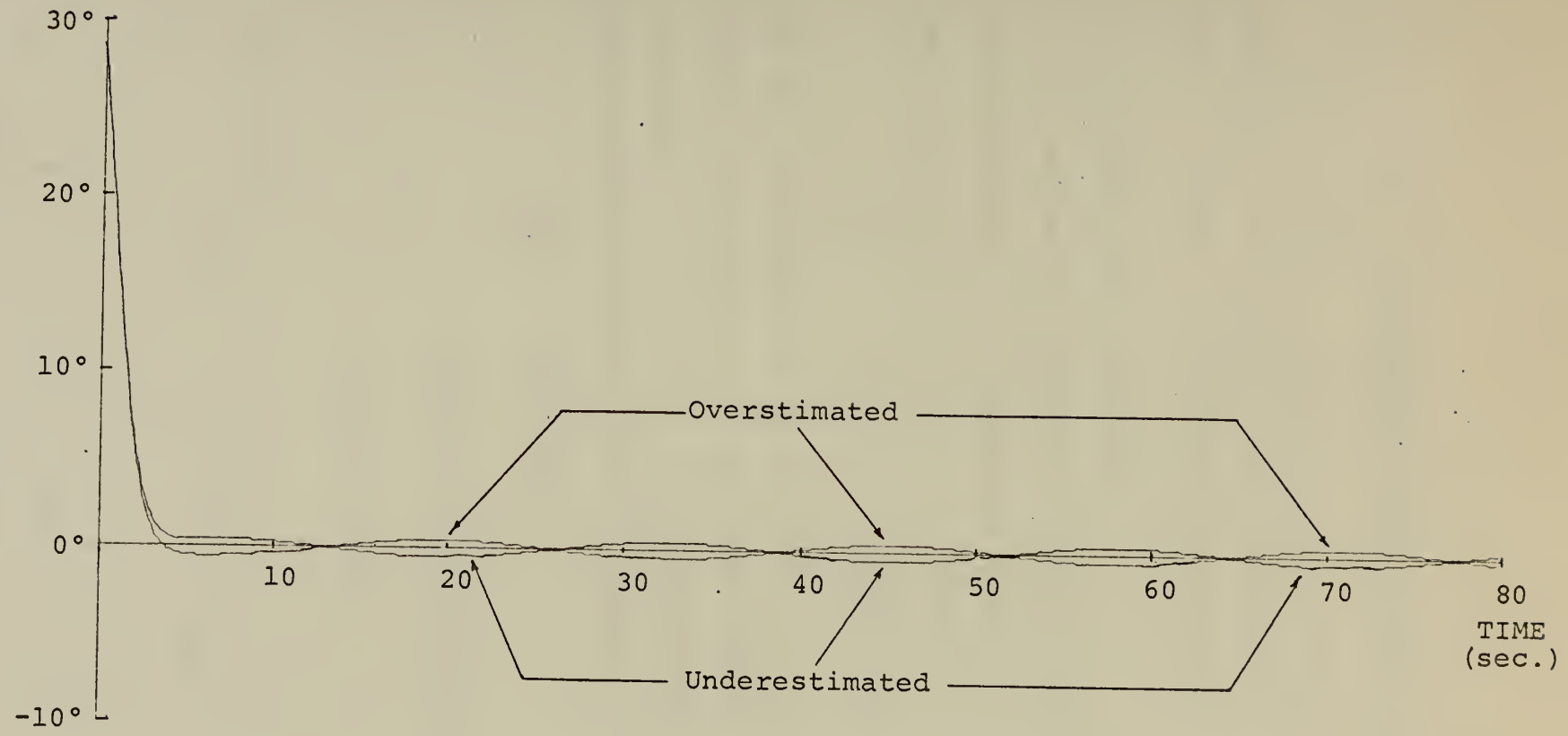

Figure 40. Pitch Response Using Suboptimal Controller
$\underline{Q} = \underline{I}$, $\underline{R} = 0.1\underline{I}$, $\hat{U} = 1.65$, $\psi = 0°$, $\hat{\lambda} = 10$, $\underline{w}(t)$ overestimated
and underestimated by 15 percent, Control Surfaces Constrained to $\pm$ 30°

VII. PRELIMINARY WORK ON WAVE ESTIMATORS

The estimators stated below are possible candidates which can provide an estimate of $\underline{w}(t)$ required for the operation of both the Optimal and Suboptimal Controllers.

A. AN INPUT-SIGNAL OBSERVER

The estimator depicted in Fig. 41 consists of a model of the plant being run in parallel with the actual plant. The state equation of the model is given by

$$\dot{\underline{e}}(t) = \underline{A}\underline{e}(t)+\underline{B}\underline{u}(t)+\underline{G}[\underline{x}(t)-\underline{e}(t)] \tag{7.1}$$

and the state equation of the plant is given by Eq. (3.9). In writing this equation, it has been assumed that all states of the system are measured. If the difference between the plant and model states $(\underline{x}(t)-\underline{e}(t))$ is called $\underline{\sigma}(t)$, then

$$\begin{aligned} \dot{\underline{\sigma}}(t) &= \dot{\underline{x}}(t)-\dot{\underline{e}}(t) \\ &= \underline{A}[\underline{x}(t)-\underline{e}(t)]-\underline{G}[\underline{x}(t)-\underline{e}(t)]+\underline{C}\underline{w}(t) \qquad (7.2) \\ &= [\underline{A}-\underline{G}]\underline{\sigma}(t)+\underline{C}\underline{w}(t) \end{aligned}$$

The solution to Eq. (7.2) is

$$\underline{\sigma}(t) = e^{[\underline{A}-\underline{G}]t}\underline{\sigma}(0)+\int_0^t e^{[\underline{A}-\underline{G}](t-\tau)}\underline{C}\underline{w}(\tau)d\tau \tag{7.3}$$

so that if the eigenvalues of $\underline{G}$ are chosen such that $e^{(\underline{A}-\underline{G})t}$ dies out quickly as t increases, then $\underline{\sigma}(t)$ can be closely approximated by

$$\underline{\sigma}(t) \simeq \int_0^t \underline{\phi}(t-\tau)\underline{C}\underline{w}(\tau)d\tau \qquad 7.4)$$

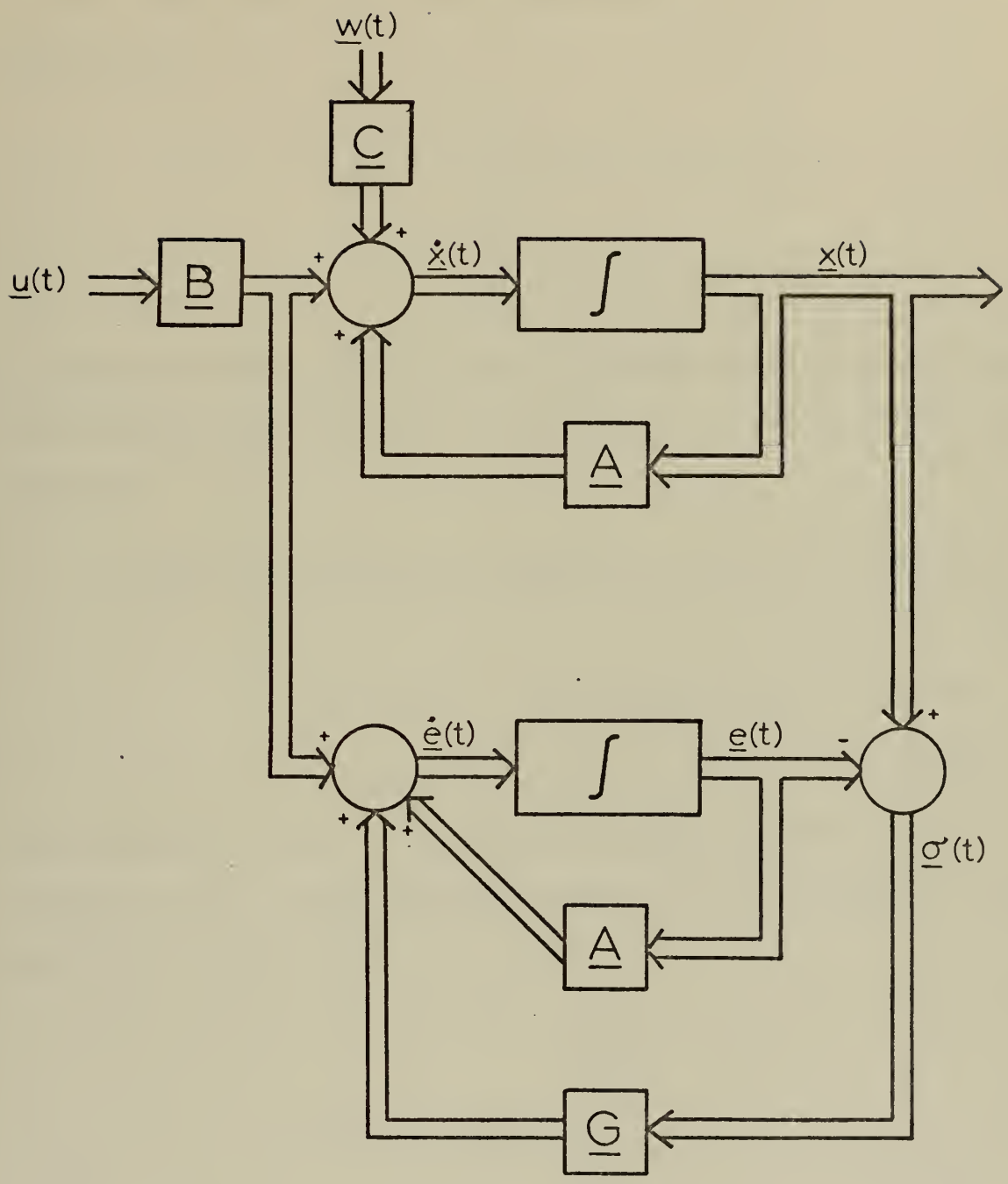

Figure 41. Input-Signal Observer

where $\underline{\phi}$ is the state transition matrix of Eq. (7.2). Referring to Eqs. (5.1) and (5.2), it is seen that Eq. (7.4) can be rewritten as

$$\underline{\sigma}(t) = \int_0^t \underline{\phi}(t-\tau) \begin{bmatrix} 0 \\ \alpha_1 \cos\omega\tau + \alpha_2 \sin\omega\tau \\ 0 \\ \beta_1 \cos\omega\tau + \beta_2 \sin\omega\tau \end{bmatrix} d\tau \tag{7.5}$$

The coefficients α_1, α_2, β_1 and β_2 are constants and can be taken outside the integral; thus, Eq. (7.5) can be rewritten in the form

$$\begin{aligned} \sigma_i(t) = {} & \alpha_1 \int_0^t \phi_{i2}(t-\tau)\cos\omega\tau d\tau + \alpha_2 \int_0^t \phi_{i2}(t-\tau)\sin\omega\tau d\tau \\ & + \beta_1 \int_0^t \phi_{i4}(t-\tau)\cos\omega\tau d\tau + \beta_2 \int_0^t \phi_{i4}(t-\tau)\sin\omega\tau d\tau \end{aligned} \tag{7.6}$$

$$(i=1,\ldots,4)$$

The integrals in Eq. (7.6) can be precomputed for a time interval $(0,t_1)$ and stored in a matrix $D(t_1)$ so that Eq. (7.6) can be put in the form

$$\underline{\sigma}(t_1) = \underline{D}(t_1) \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta 2 \end{bmatrix} \tag{7.7}$$

If the state differences are now sampled and stored at times t_i, $i=1, \ldots, k$, then an estimate of the coefficients α_1, α_2, β_1 and β_2 can be obtained by determining the minimum-mean-square solution of

$$\begin{bmatrix} \underline{\sigma}(t_1) \\ \underline{\sigma}(t_2) \\ \vdots \\ \underline{\sigma}(t_k) \end{bmatrix} = \begin{bmatrix} \underline{D}(t_1) \\ \underline{D}(t_2) \\ \vdots \\ \underline{D}(t_k) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \equiv \underline{\mathcal{D}} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{7.8}$$

given by

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \underline{\mathcal{D}}^{\#} \begin{bmatrix} \underline{\sigma}(t_1) \\ \underline{\sigma}(t_2) \\ \vdots \\ \underline{\sigma}(t_k) \end{bmatrix} \tag{7.9}$$

where $\underline{\mathcal{D}}^{\#}$ is the pseudo-inverse of $\underline{\mathcal{D}}$.

B. FAST-FOURIER TRANSFORM ESTIMATION

An alternative approach is to Laplace transform Eq. (7.2) to give

$$\begin{aligned} \underline{\sigma}(s) &= s[s\underline{I}-\underline{A}+\underline{G}]^{-1}\underline{\sigma}(0) \\ &\quad +[s\underline{I}-\underline{A}+\underline{G}]^{-1}\underline{C}\underline{w}(s) \end{aligned} \tag{7.10}$$

If $\underline{w}(t)$ is a periodic function, then the spectrum $\underline{\sum}(f)$ will be

$$\underline{\sum}(f) = [j2\pi f\underline{I}-\underline{A}+\underline{G}]^{-1}\underline{C}\underline{W}(f) \tag{7.11}$$

and an estimate of $\underline{W}(f)$ can be determined by

$$\underline{C}\underline{W}(f) = [j2\pi fI-A+G]\sum(f) \tag{7.12}$$

where $\sum(f)$ can be determined by taking the Fast-Fourier Transform (FFT) of the error between the predicted and actual responses.

C. STATE OBSERVER MODEL

A third possibility is to expand the state equation given by Eq. (3.9) by defining four additional states. The matrix product

$$\underline{C}\underline{w}(t) = \begin{bmatrix} 0 \\ \alpha_1 \cos\omega t + \alpha_2 \sin\omega t \\ 0 \\ \alpha_1 \cos\omega t + \beta_2 \sin\omega t \end{bmatrix} \tag{7.13}$$

can be rewritten as

$$\underline{C}\underline{w}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & \alpha_1 \\ \cos\omega t & \sin\omega t & 0 & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & \beta_1 \\ 0 & 0 & \cos\omega t & \sin\omega t & \beta_2 \end{bmatrix} \tag{7.14}$$

The augmented state vector is now defined as

$$\underline{x}(t) = \begin{bmatrix} \hat{z}(t) \\ \dot{\hat{z}}(t) \\ \theta(t) \\ \dot{\theta}(t) \\ \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{7.15}$$

so that the state equation becomes

$$\dot{\underline{x}}(t) = \left[\begin{array}{c|cccc} \underline{A} & 0 & 0 & 0 & 0 \\ & \cos\omega t & \sin\omega t & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & 0 & 0 & \cos\omega t & \sin\omega t \\ \hline \underline{0} & & & \underline{0} & \end{array}\right] \underline{x}(t) + \left[\begin{array}{c} \underline{B} \\ \hline \underline{0} \end{array}\right] \underline{u}(t)$$

(7.16)

$$= \underline{A}'(t)\underline{x}(t)+\underline{B}'\underline{u}(t)$$

The actual physical states ($x_1(t)$ through $x_4(t)$) are measured. The observation matrix for the augmented system is

$$\underline{H} = [\underline{I} \quad \underline{0}] \tag{7.17}$$

If an observer, similar to Fig. 41 but with eight states and an observation matrix $\underline{H}$, is now run in parallel with the actual plant so that the observer's state equation is

$$\dot{\underline{e}}(t) = \underline{A}'(t)\underline{e}(t)+\underline{B}'\underline{u}(t)+\underline{G}(t)[\underline{H}\underline{e}(t)-\underline{H}\underline{x}(t)] \tag{7.18}$$

then the difference between the observer and actual state equations can be written as

$$\dot{\underline{\sigma}}(t) = [\underline{A}'(t)+\underline{G}(t)\underline{H}]\underline{\sigma}(t) \tag{7.19}$$

By choosing $\underline{G}(t)$ so that $[\underline{A}'(t)+\underline{G}(t)\underline{H}]$ is stable, then the states of the observer, $\underline{e}(t)$, will approach the states $\underline{x}(t)$ and the error $\underline{\sigma}(t)$ will approach zero. The states $e_5(t)$ through $e_8(t)$ will then be estimates of α_1, α_2, β_1 and β_2 respectively.

Utilizing the above method, an example can be given for the first-order case where

$$\dot{x}(t) = -ax(t)+\alpha\cos\omega t \qquad a>0$$

The states are augmented by defining x_2 so that

$$\dot{x}_1(t) = -ax_1+(\cos t)x_2$$

$$\dot{x}_2(t) = 0$$

or

$$\dot{\underline{x}}(t) = \underline{A}(t)\underline{x}(t)$$

The observer state equation is

$$\dot{\underline{e}}(t) = \underline{A}(t)\underline{e}(t)+\underline{G}(t)\underline{H}[\underline{e}(t)-\underline{x}(t)]$$

where the observation matrix is

$$\underline{H} = [1 \quad 0]$$

The matrix

$$[\underline{A}(t)+\underline{G}(t)\underline{H}] = \begin{bmatrix} -a+g_1 & \cos\omega t \\ g_2 & 0 \end{bmatrix}$$

must now be tested for stability which can be accomplished by defining a Lyapunov function [Ref. 8]

$$V(t) \equiv 1/2\ (\sigma_1^2(t)+\sigma_2^2(t)) \tag{7.20}$$

and its time derivative

$$\dot{V}(t) = \sigma_1(t)\dot{\sigma}_1(t)+\sigma_2(t)\dot{\sigma}_2(t) \tag{7.21}$$

For this example,

$$\dot{V}(t) = (-a+g_1)\sigma_1^2(t)+(\cos\omega t)\sigma_1(t)\sigma_2(t)+g_2\sigma_1(t)\sigma_2(t)$$

$$= (-a+g_1)\sigma_1^2(t)+(\cos\omega t+g_2)\sigma_1(t)\sigma_2(t)$$

where if the values of g_1 and g_2 are chosen such that

$$g_1 < a$$

$$g_2 = -\cos\omega t$$

then Eq. (7.20) is positive definite and $\dot{v}(t)<0$. The system is therefore stable and the estimator states will approach the plant states as t approaches infinity.

If this approach is used, additional work will be required to ensure that $\underline{G}(t)$ can be selected so that $[\underline{A}'(t)+\underline{G}(t)\underline{H}]$ is stable for the ship.

VIII. CONCLUSIONS

A. THE CONTROLLER

The easiest controller to implement is the Constant-Gain Controller since, as previously stated, it in no way depends upon any knowledge (past, present or future) of the wave forces and moments. This controller would also be the least expensive since the other two controllers are essentially the Constant-Gain Controller with additional computed control signals. On the other hand, both the Optimal Controller and the Suboptimal Controller would be more expensive and complex than the Constant-Gain Controller but would maintain the state vector closer to the origin.

The complexity of realizing the command signal depends on the nature of the wave forces and how they are estimated. If $\underline{w}(t)$ is sinusoidal in form and is estimated directly in the time domain, $\underline{v}(t)$ can be obtained directly for the Suboptimal Controller whereas the Optimal Controller requires either on-line or precomputed solutions of Eq. (4.25). In this case, the Suboptimal Controller is considered to be less complex and the "best" controller. If, however, $\underline{w}(t)$ is some general periodic function requiring the FFT estimator or if the frequency domain approach is used for the single sinusoidal $\underline{w}(t)$, then the difference in complexity between the Optimal and Suboptimal Controllers is negligible and the choice of the "best" controller will depend upon how close it is desired to maintain

the state vector to the origin. In either case, the estimator output could be set to zero (if the estimates were bad) and both controllers would reduce to the Constant-Gain Controller, resulting in still satisfactory performance.

B. RECOMMENDATIONS FOR FURTHER STUDY

The assumptions used in this investigation concerning sea conditions, control surface deflections and measurements result in the recommendations given below.

1. Sea Conditions

The constant height-to-wavelength ratio and single-frequency wave used in this investigation is not indicative of the true sea condition which is a spectrum of different height-to-wavelength ratios and frequencies [Ref. 2]. A more realistic model for sea conditions should be derived and the ship's response to this model investigated.

2. Control Dynamics

The ship's response should be investigated for the case where the dynamics of the control surface actuation system are included. This investigation assumed an instantaneous control surface deflection capability.

3. Measurement Noise

The ideal case of zero measurement noise was assumed throughout this investigation. Further investigation is recommended to include realistic measurement noise.

4. Wave Estimator

A wave estimator should be investigated for inclusion in the ship model and the response determined using the estimator and measurement noises as noted above.

5. Weighting Matrices

Further investigation is recommended in the determination of the values of the weighting matrices $\underline{Q}$ and $\underline{R}$. The possibility of exceeding control surface or structural rates and the accompanying effects on personnel was not included in this investigation. If further investigation proves this to be a problem area, then new values for the state and control weights will have to be determined.

6. Wave Contouring

The inclusion of a realistic sea condition model will change the point where upper-body contact with the wave occurs, previously stated as being wavelengths greater than 14. This point of contact should be determined and a controller that would allow the ship to follow the wave contour instead of driving through the wave should be utilized for this case. This would result in a linear tracking problem instead of the linear regulator approach used in this investigation.

APPENDIX A

Here it is shown that the matrix equality

$$\underline{B}'\underline{v}(t) = -\underline{C}'\underline{w}(t) \tag{4.29}$$

holds, where

$$\underline{B}' = \begin{bmatrix} b_{21} & b_{22} \\ b_{41} & b_{42} \end{bmatrix}$$

$$\underline{C}' = \begin{bmatrix} c_{21} & c_{22} \\ c_{41} & c_{42} \end{bmatrix}$$

and from which the command signal, $\underline{v}(t)$, can be obtained as

$$\underline{v}(t) = -\underline{B}'^{-1}\underline{C}'\underline{w}(t) \tag{4.30}$$

The matrix product

$$\underline{B}'\underline{v}(t) = \begin{bmatrix} b_{21} & b_{22} \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

$$= \begin{bmatrix} b_{21}v_1(t)+b_{22}v_2(t) \\ b_{41}v_1(t)+b_{42}v_2(t) \end{bmatrix}$$

and

$$-\underline{C}'\underline{w}(t) = -\begin{bmatrix} C_{21} & C_{22} \\ C_{41} & C_{42} \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

$$= -\begin{bmatrix} C_{21}w_1(t)+C_{22}w_2(t) \\ C_{41}w_1(t)+C_{42}w_2(t) \end{bmatrix}$$

Therefore, since it is given by Eq. (4.27) that

$$b_{21}v_1(t)+b_{22}v_2(t) = -(C_{21}w_1(t)+C_{22}w_2(t))$$

and

$$b_{41}v_1(t)+b_{42}v_2(t) = -(C_{41}w_1(t)+C_{42}w_2(t))$$

which can be written as

$$\begin{bmatrix} b_{21}v_1(t)+b_{22}v_2(t) \\ b_{41}v_1(t)+b_{42}v_2(t) \end{bmatrix} = -\begin{bmatrix} C_{21}w_1(t)+C_{22}w_2(t) \\ C_{41}w_1(t)+C_{42}w_2(t) \end{bmatrix}$$

or,

$$\underline{B}'\underline{v}(t) = -\underline{C}'\underline{w}(t)$$

APPENDIX B

Here it is shown that the matrix equality

$$\underline{B}\underline{G} = -\underline{C} \tag{4.33}$$

holds, which reduces the system equation to

$$\dot{\underline{x}} = [\underline{A}+\underline{B}\underline{F}(t)]\underline{x}(t), \tag{4.5}$$

a stable system whose state approaches zero as t approaches infinity, when the suboptimal controller is used.

$$\underline{B}\underline{G} = \underline{B}(-\underline{B}'^{-1}\underline{C}')$$

$$= -\underline{B}\underline{B}'^{-1}\underline{C}'$$

$$= -\begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix} \frac{\begin{bmatrix} b_{42} & -b_{22} \\ -b_{41} & b_{21} \end{bmatrix}}{(b_{21}b_{42}-b_{22}b_{41})} \begin{bmatrix} c_{21} & c_{22} \\ c_{41} & c_{42} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{21} & c_{22} \\ c_{41} & c_{42} \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & 0 \\ c_{21} & c_{22} \\ 0 & 0 \\ c_{41} & c_{42} \end{bmatrix}$$

$$= -\underline{C}$$

SEMISUBMERGED SHIP PROGRAM

THIS PROGRAM SIMULATES A SEMISUBMERGED SHIP AND COMPUTES THE LONGITUDINAL RESPONSE FOR EITHER AHEAD OR FOLLOWING SEAS.

THE USER HAS THE CHOICE OF THREE CONTROL SYSTEMS--

SUBOPTIMAL
CONSTANT-GAIN
OPTIMAL

OR EITHER NONE AT ALL.

A MAXIMUM OF TWO (2) RUNS CAN BE MADE EACH TIME TO ALLOW A COMPARISON TO BE MADE IF ANY OF THE PARAMETERS ON THE DATA CARDS LISTED BELOW ARE CHANGED.

DATA CARDS--

1. NUMBER OF RUNS (1 OR 2)
2. SHIP'S NON-DIMENSIONAL SPEED (UHAT)
3. TIME PARAMETERS --INITIAL TIME (TI), TIME INCREMENT (DT), FINAL TIME (TF)
4. Q MATRIX--ROW #1
5. Q MATRIX--ROW #2
6. Q MATRIX--ROW #3
7. Q MATRIX--ROW #4
8. R MATRIX--ROW #1
9. R MATRIX--ROW #2
10. CONTROL OR NO CONTROL--
    >0 => YES
    =0 => NO

IF NO CONTROL SYSTEM IS USED, DATA CARDS 11 & 12 ARE NOT USED

11. TYPE OF CONTROL SYSTEM--

```
                                        -1 => SUBOPTIMAL
                                         0 => CONSTANT-GAIN
                                        +1 => OPTIMAL

        12. UNSATURATED OR SATURATED CONTROLS--
                                                  -1 => UNSATURATED
                                                  +1 => SATURATED

        13. SHIP'S NON-DIMENSIONAL INITIAL CONDITIONS--
                                                        HEAVE
                                                        HEAVE VELOCITY
                                                        PITCH
                                                        PITCH VELOCITY
        14. SEA CONDITION--
                                 -1 => FOLLOWING SEA
                                  0 => CALM SEA
                                 +1 => AHEAD SEA

        15. NON-DIMENSIONAL OCEAN WAVELENGTH
```

IF TWO (2) RUNS ARE CALLED FOR IN DATA CARD #1, THEN DATA CARDS 16--29 WILL HAVE THE SAME FORMAT AS CARDS 2--15.

```
// EXEC FORTCLGP,REGION.GO=166K
//FORT.SYSIN DD *
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH000660
      REAL*8 JU,JX,JT,MHATW                                             SH000670
      COMMON A(4,4),B(4,2),F(2,4),P(4,4),Q(4,4),RIBT(2,4),TEST,TEMP1(10)SH000680
     *,R(2,2),G(2,2),DEG,U(2,1),JU,JX,JT,MHATW,ZHATW,WEFRQ,A1,B1,D1,E1, SH000690
     *A2,B2,D2,E2,KNTRL,KSAT,NKNTRL,NK,II,JJ,LL                          SH000700
      DIMENSION PRMT(5),Y(14),DERY(14),AUX(16,14),BT(2,4),BP(2,2),      SH000710
     *BPI(2,2),CP(2,2),GM(2,2),RI(2,2)                                  SH000720
      EXTERNAL FCT,OUTP                                                 SH000730
      PI=3.1415927D0                                                    SH000740
      DEG=180.D0/PI                                                     SH000750
**
**  READ NUMBER OF RUNS TO BE MADE
**
      READ(5,1452) NCURV                                                SH000760
      NK=NCURV                                                          SH000770
   11 JU=0.D0                                                           SH000800
      JX=0.D0                                                           SH000810
      JT=0.D0                                                           SH000820
**
**  READ SHIP'S NON-DIMENSIONAL SPEED
**
```

```
      READ(5,1000) UHAT                                                 SH000830
      WRITE(6,1010) UHAT                                                SH000840
**
**    COMPUTE THE A MATRIX
**
      A(1,1)=0.D0                                                       SH000850
      A(1,2)=1.D0                                                       SH000860
      A(1,3)=0.D0                                                       SH000870
      A(1,4)=0.D0                                                       SH000880
      A(2,1)=-.2951D0                                                   SH000890
      A(2,2)=-.9362D0*UHAT                                              SH000900
      A(2,3)=-.9362D0*UHAT**2+.207D-1                                   SH000910
      A(2,4)=-.12990D1*UHAT                                             SH000920
      A(3,1)=0.D0                                                       SH000930
      A(3,2)=0.D0                                                       SH000940
      A(3,3)=0.D0                                                       SH000950
      A(3,4)=1.D0                                                       SH000960
      A(4,1)=.12D-1                                                     SH000970
      A(4,2)=-.5742D0*UHAT                                              SH000980
      A(4,3)=-.5742D0*UHAT**2-.2961D0                                   SH000990
      A(4,4)=-.2091D1                                                   SH001000
      II=0                                                              SH001010
      JJ=0                                                              SH001020
      LL=0                                                              SH001030
**
**    COMPUTE THE B MATRIX
**
      DO 1 J=1,2                                                        SH001040
      B(1,J)=0.D0                                                       SH001050
      B(3,J)=0.D0                                                       SH001060
    1 CONTINUE                                                          SH001070
      B(2,1)=-.2964D0*UHAT**2                                           SH001130
      B(2,2)=-.1624D0*UHAT**2                                           SH001140
      B(4,1)=-.3706D0*UHAT**2                                           SH001150
      B(4,2)=.1432D0*UHAT**2                                            SH001160
**
**    READ TIME PARAMETERS--TI,DT,TF
**
   12 READ(5,1100) TI,DT,TF                                             SH001320
      WRITE(6,1105) TI,DT,TF                                            SH001330
      TEST=TF/(4.D0*DT)                                                 SH001340
      PRMT(1)=TF                                                        SH001350
      PRMT(2)=TI                                                        SH001360
      PRMT(3)=-DT                                                       SH001370
      PRMT(4)=1.D-5                                                     SH001380
      DO 2 I=1,10                                                       SH001080
      TEMP1(I)=0.D0                                                     SH001090
      Y(I)=0.D0                                                         SH001100
```

```
      DERY(I)=.1D0                                                      SH001110
    2 CONTINUE                                                          SH001120
      WRITE(6,1110)                                                     SH001390
**
** READ Q MATRIX ROW BY ROW
**
      DO 3 I=1,4                                                        SH001400
      READ(5,1200) (Q(I,J),J=1,4)                                       SH001410
      WRITE(6,1111) (Q(I,J),J=1,4)                                      SH001420
    3 CONTINUE                                                          SH001430
      WRITE(6,1210)                                                     SH001440
**
** READ R MATRIX ROW BY ROW
**
      DO 4 I=1,2                                                        SH001450
      READ(5,1112) (R(I,J),J=1,2)                                       SH001460
      WRITE(6,1205) (R(I,J),J=1,2)                                      SH001470
    4 CONTINUE                                                          SH001480
**
** READ WHETHER CONTROL SYSTEM IS DESIRED OR NOT
**
      READ(5,1452) NKNTRL                                               SH001162
      IF(NKNTRL.EQ.0) GO TO 13                                          SH001485
      WRITE(6,1005)                                                     SH001530
**
** COMPUTE THE CONSTANT K MATRIX
**
      CALL DTRAN(B,BT,4,2)                                              SH001540
      CALL DRINV(R,RI)                                                  SH001550
      CALL DMPRD(RI,BT,RIBT,2,2,4)                                      SH001560
      CALL DMPRD(B,RIBT,P,4,2,4)                                        SH001570
      CALL DHPCG(PRMT,Y,DERY,10,IHLF,FCT,OUTP,AUX)                      SH001580
      WRITE(6,1300) IHLF                                                SH001590
      LL=4                                                              SH001600
**
** READ TYPE OF CONTROL SYSTEM TO USE
**
      READ(5,1451) KNTRL                                                SH001770
**
** READ WHETHER OR NOT THE CONTROLS ARE SATURATED
**
      READ(5,1451) KSAT                                                 SH001775
      IF (KNTRL) 8,9,10                                                 SH001780
    8 WRITE(6,1465)                                                     SH001790
**
** COMPUTE THE G MATRIX
**
      CP(1,1)=0.4907D0                                                  SH001170
```

```
      CP(1,2)=-0.503D-1                                                 SH001180
      CP(2,1)=-0.502D-1                                                 SH001190
      CP(2,2)=0.2865D0                                                  SH001200
      DO 6 I=1,2                                                        SH001210
      K=2*I                                                             SH001220
      DO 6 J=1,2                                                        SH001230
      BP(I,J)=B(K,J)                                                    SH001240
    6 CONTINUE                                                          SH001250
      CALL DRINV(BP,BPI)                                                SH001260
      CALL DMPRD(BPI,CP,GM,2,2,2)                                       SH001270
      DO 7 I=1,2                                                        SH001280
      DO 7 J=1,2                                                        SH001290
      G(I,J)=-GM(I,J)                                                   SH001300
    7 CONTINUE                                                          SH001310
      GO TO 121                                                         SH001800
    9 WRITE(6,1466)                                                     SH001810
      GO TO 121                                                         SH001820
   10 WRITE(6,1467)                                                     SH001830
      M=2                                                               SH001490
      IF(R(1,1).LT.0.9D0) M=0                                           SH001500
      IF(R(1,1).LT.0.4D0) M=-2                                          SH001510
      CALL EST(A2,B2,E2,D2,M)                                           SH001520
      GO TO 121                                                         SH001832
  121 PRMT(1)=TI                                                        SH001610
      PRMT(2)=TF                                                        SH001620
      PRMT(3)=DT                                                        SH001630
      DO 5 I=1,6                                                        SH001640
      DERY(I)=1.D0/6.D0                                                 SH001650
    5 CONTINUE                                                          SH001660
      WRITE(6,1250)                                                     SH001670
**
**  READ SHIP'S NON-DIMENSIONAL INITIAL CONDITIONS
**
   13 READ(5,1200) (Y(I),I=1,4)                                         SH001680
      WRITE(6,1350) UHAT,(Y(I),I=1,4)                                   SH001690
      Y(5)=0.D0                                                         SH001700
      Y(6)=0.D0                                                         SH001710
      II=0                                                              SH001720
      JJ=0                                                              SH001730
      IF(NKNTRL.EQ.0) WRITE(6,1468)                                     SH001734
**
**  READ SEA CONDITION
**
      READ(5,1451) ISEA                                                 SH001740
      IF(ISEA.EQ.0.D0) GO TO 119                                        SH001750
**
**  READ NON-DIMENSIONAL OCEAN WAVELENGTH
**
```

```
      READ(5,1461) WLHAT                                                SH001760
**
**   COMPUTE THE WAVE FORCES (ZHATW) AND MOMENTS (MHATW)
**
      IF(ISEA) 100,119,110                                              SH001840
**
**   FOLLOWING SEA
**
  100 PSI=0.D0                                                          SH001850
      WRITE(6,1470) WLHAT                                               SH001860
      WEFRQ=DSQRT(2.D0*PI/WLHAT)-2.D0*PI*UHAT/WLHAT*DCOS(PSI/DEG)       SH001870
      IF(UHAT.GT.1.1D0) GO TO 101                                       SH001880
      A1=0.408926D0-0.294982D0*WLHAT+0.591889D-1*WLHAT**2-0.510023D-2*  SH001890
     *   WLHAT**3+0.2318D-3*WLHAT**4-0.539434D-5*WLHAT**5+0.505979D-7*  SH001900
     *   WLHAT**6                                                       SH001910
      B1=-0.350088D0+0.147652D0*WLHAT-0.164184D-1*WLHAT**2+0.887811D-3* SH001920
     *   WLHAT**3-0.254773D-4*WLHAT**4+0.3331D-6*WLHAT**5-0.109496D-8*  SH001930
     *   WLHAT**6                                                       SH001940
      D1=-1.39117D0+0.597654D0*WLHAT-0.768856D-1*WLHAT**2+0.539146D-2*  SH001950
     *   WLHAT**3-0.214538D-3*WLHAT**4+0.451631D-5*WLHAT**5-0.389348D-7*SH001960
     *   WLHAT**6                                                       SH001970
      E1=0.388758D0-0.274848D0*WLHAT+0.550417D-1*WLHAT**2-0.465091D-2*  SH001980
     *   WLHAT**3+0.204644D-3*WLHAT**4-0.456401D-5*WLHAT**5+0.407234D-7*SH001990
     *   WLHAT**6                                                       SH002000
      GO TO 120                                                         SH002010
  101 A1=0.626065D0-0.453279D0*WLHAT+0.91014D-1*WLHAT**2-0.776281D-2*   SH002020
     *   WLHAT**3+0.348576D-3*WLHAT**4-0.803407D-5*WLHAT**5+0.750506D-7*SH002030
     *   WLHAT**6                                                       SH002040
      B1=-0.507524D0+0.222259D0*WLHAT-0.273648D-1*WLHAT**2+0.170222D-2* SH002050
     *   WLHAT**3-0.576673D-4*WLHAT**4+0.966982D-6*WLHAT**5-0.584761D-8*SH002060
     *   WLHAT**6                                                       SH002070
      D1=-2.33443D0+0.971726D0*WLHAT-0.12184D0*WLHAT**2+0.830526D-2*    SH002080
     *   WLHAT**3-0.32266D-3*WLHAT**4+0.6696D-5*WLHAT**5-0.575847D-7*   SH002090
     *   WLHAT**6                                                       SH002100
      E1=0.766135D0-0.492584D0*WLHAT+0.100878D0*WLHAT**2-0.900991D-2*   SH002110
     *   WLHAT**3+0.424255D-3*WLHAT**4-0.102077D-4*WLHAT**5+0.988452D-7*SH002120
     *   WLHAT**6                                                       SH002130
      GO TO 120                                                         SH002140
**
**   AHEAD SEA
**
  110 PSI=180.D0                                                        SH002150
      WRITE(6,1475) WLHAT                                               SH002160
      WEFRQ=DSQRT(2.D0*PI/WLHAT)-2.D0*PI*UHAT/WLHAT*DCOS(PSI/DEG)       SH002170
      IF(UHAT.GT.1.1D0) GO TO 111                                       SH002180
      A1=0.502667D0-0.364173D0*WLHAT+0.798172D-1*WLHAT**2-0.766724D-2*  SH002190
     *   WLHAT**3+0.385676D-3*WLHAT**4-0.983189D-5*WLHAT**5+0.100043D-6*SH002200
     *   WLHAT**6                                                       SH002210
```

```
      B1=-0.547692D-1+0.662901D-2*WLHAT+0.12247D-2*WLHAT**2-0.245175D-3*SH002220
     *    WLHAT**3+0.144963D-4*WLHAT**4-0.395096D-6*WLHAT**5+0.416539D-8*SH002230
     *    WLHAT**6                                                       SH002240
      D1=1.04556D0-0.442992D0*WLHAT+0.528556D-1*WLHAT**2-0.33672D-2*     SH002250
     *    WLHAT**3+0.119255D-3*WLHAT**4-0.218464D-5*WLHAT**5+0.15902D-7* SH002260
     *    WLHAT**6                                                       SH002270
      E1=-0.551822D0+0.224035D0*WLHAT-0.290222D-1*WLHAT**2+0.227802D-2* SH002280
     *    WLHAT**3-0.104387D-3*WLHAT**4+0.256714D-5*WLHAT**5-0.259431D-7*SH002290
     *    WLHAT**6                                                       SH002300
      GO TO 120                                                          SH002310
  111 A1=0.714136D0-0.531282D0*WLHAT+0.11505D0*WLHAT**2-0.107728D-1*     SH002320
     *    WLHAT**3+0.529087D-3*WLHAT**4-0.13218D-4*WLHAT**5+0.13215D-6*  SH002330
     *    WLHAT**6                                                       SH002340
      B1=0.155114D0-0.964082D-1*WLHAT+0.163468D-1*WLHAT**2-0.127389D-2* SH002350
     *    WLHAT**3+0.472894D-4*WLHAT**4-0.787404D-6*WLHAT**5+0.405063D-8*SH002360
     *    WLHAT**6                                                       SH002370
      D1=1.81165D0-0.776761D0*WLHAT+0.949809D-1*WLHAT**2-0.62921D-2*     SH002380
     *    WLHAT**3+0.237888D-3*WLHAT**4-0.483654D-5*WLHAT**5+0.411761D-7*SH002390
     *    WLHAT**6                                                       SH002400
      E1=-0.45764D0+0.11325D0*WLHAT+0.23954D-2*WLHAT**2-0. 15712D-2*     SH002410
     *    WLHAT**3+0.854828D-4*WLHAT**4-0.263091D-5*WLHAT**5+0.299181D-7*SH002420
     *    WLHAT**6                                                       SH002430
      GO TO 120                                                          SH002440
**
**   CALM SEA
**
  119 WRITE(6,1480)                                                      SH002450
      A1=0.D0                                                            SH002460
      B1=0.D0                                                            SH002470
      D1=0.D0                                                            SH002480
      E1=0.D0                                                            SH002490
      A2=0.D0                                                            SH002500
      B2=0.D0                                                            SH002510
      D2=0.D0                                                            SH002520
      E2=0.D0                                                            SH002530
      WEFRQ=0.D0                                                         SH002540
**
**   COMPUTE THE TIME HISTORIES OF THE SHIP'S RESPONSES
**
  120 CALL DHPCG (PRMT,Y,DERY,6,IHLF,FCT,OUTP,AUX)                       SH002550
      WRITE(6,1300) IHLF                                                 SH002560
      WRITE(6,1485) JX,JU,JT                                             SH002570
      NK=NK-1                                                            SH002580
**
**   RETURN FOR MORE DATA IF ANOTHER RUN IS CALLED FOR
**
      IF(NK.GT.0) GO TO 11                                               SH002590
      LL=10                                                              SH002600
```

```
**
**  PRINT OR PLOT OUTPUT AS DESIRED
**
      CALL OUTP(X,Y,DERY,IHLF,NDIM,PRMT)                                SH002610
 1000 FORMAT(D20.10)                                                    SH002620
 1005 FORMAT(/,/,/,/,/,/)                                               SH002630
 1010 FORMAT('1',T20,'SHIP SPEED =',D16.8,/,/,/)                        SH002640
 1100 FORMAT(3D20.10)                                                   SH002650
 1105 FORMAT(T20,'INITIAL TIME    =',D16.8,/,T20,'TIME INCREMENT =',D16.8SH002660
     *       ,/,T20,'FINAL TIME      =',D16.8)                          SH002670
 1110 FORMAT(/,/,T40,'Q MATRIX',/,/)                                    SH002680
 1111 FORMAT('0',4(4X,D16.8))                                           SH002690
 1112 FORMAT(2D20.10)                                                   SH002700
 1200 FORMAT(4D20.10)                                                   SH002710
 1205 FORMAT('0',20X,2(4X,D16.8))                                       SH002720
 1210 FORMAT(/,/,/,T40,'R MATRIX',/,/)                                  SH002730
 1250 FORMAT('1')                                                       SH002740
 1300 FORMAT(/,/,T20,'IHLF =',I2)                                       SH002750
 1350 FORMAT(/,/,/,/,T20,'SHIP INITIAL CONDITIONS :',/,T40,'SHIP SPEED',SH002760
     *      T54,' =',D16.8,/,/,T40,'HEAVE',T54,' =',D16.8,/,/,T40,      SH002770
     *      'HEAVE VELOCITY =',D16.8,/,/,T40,'PITCH',T54,' =',D16.8,/,/,SH002780
     *      T40,'PITCH VELOCITY =',D16.8)                               SH002790
 1451 FORMAT(I2)                                                        SH002800
 1452 FORMAT(I1)                                                        SH002810
 1461 FORMAT(D20.10)                                                    SH002820
 1465 FORMAT(/,/,T20,'CONTROL SYSTEM---SUBOPTIMAL',/,'+',T37,'__________SH002830
     *')                                                                SH002840
 1466 FORMAT(/,/,T20,'CONTROL SYSTEM---CONSTANT GAIN',/,'+',T37,'_______SH002850
     *______')                                                          SH002860
 1467 FORMAT(/,/,T20,'CONTROL SYSTEM---OPTIMAL',/,'+',T37,'_______')    SH002870
 1468 FORMAT(/,/,T20,'CONTROL SYSTEM---NONE',/,'+',T37,'____')          SH002875
 1470 FORMAT(/,/,T20,'SEA CONDITIONS:',/,T35,'(1) FOLLOWING SEA',/,T35, SH002880
     *       '(2) WAVELENGTH =',D16.8)                                  SH002890
 1475 FORMAT(/,/,T20,'SEA CONDITIONS:',/,T35,'(1) AHEAD SEA',/,T35,     SH002900
     *       '(2) WAVELENGTH =',D16.8)                                  SH002910
 1480 FORMAT(/,/,T20,'SEA CONDITIONS------PERFECTLY CALM')              SH002920
 1485 FORMAT(/,/,T58,'COST FUNCTIONALS',/,'+',T58,'_________________',/,/SH002921
     *       ,T59,'T',/,T33,'JX = 0.5*INTEGRAL(T0,TF)(X QX)DT = ',D16.8,SH002922
     *       /,/,T59,'T',/,T33,'JU = 0.5*INTEGRAL(T0,TF)(U RU)DT = ',   SH002923
     *       D16.8,/,/,T54,'J = JX + JU = ',D16.8)                      SH002924
      STOP                                                              SH002930
      END                                                               SH002940
```

```
      SUBROUTINE DTRAN(X,Y,M,N)                                         SH002980
**
**    THIS SUBROUTINE TAKES THE TRANSPOSE OF AN (M X N) MATRIX X AND RETURNS IT
**    IN AN (N X M) MATRIX Y.   (DOUBLE PRECISION)
**
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH002990
      DIMENSION X(M,N),Y(N,M)                                           SH003000
      DO 1 I=1,M                                                        SH003010
      DO 1 J=1,N                                                        SH003020
      Y(J,I)=X(I,J)                                                     SH003030
    1 CONTINUE                                                          SH003040
      RETURN                                                            SH003050
      END                                                               SH003060

      SUBROUTINE DMPRD(A,B,C,L,M,N)                                     SH003100
**
**    THIS SUBROUTINE PERFORMS THE DOUBLE PRECISION MATRIX MULTIPLICATION
**    A(L X M)*B(M X N)=C(L X N)
**
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH003110
      DIMENSION A(L,M),B(M,N),C(L,N)                                    SH003120
      DO 1 I=1,L                                                        SH003130
      DO 1 J=1,N                                                        SH003140
      C(I,J)=0.D0                                                       SH003150
      DO 1 K=1,M                                                        SH003160
    1 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                       SH003170
      RETURN                                                            SH003180
      END                                                               SH003190

      SUBROUTINE FCT(X,Y,DERY)                                          SH003230
**
**    THIS SUBROUTINE CONTAINS THE DIFFERENTIAL EQUATIONS TO BE SOLVED BY THE IBM
**    SUBROUTINE DHPCG.
**
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH003240
      REAL*8 JU,JX,JT,MHATW                                             SH003250
      COMMON A(4,4),B(4,2),F(2,4),P(4,4),Q(4,4),RIBT(2,4),TEST,TEMP1(10)SH003260
     *,R(2,2),G(2,2),DEG,U(2,1),JU,JX,JT,MHATW,ZHATW,WEFRQ,A1,B1,D1,E1, SH003270
     *A2,B2,D2,E2,KNTRL,KSAT,NKNTRL,NK,II,JJ,LL                          SH003280
      DIMENSION XT(1,4),QX(4),UT(1,2),RU(2,1),V(2),Y(1),DERY(1)         SH003290
      IF(LL.GT.2) GO TO 2                                               SH003300
**
**    DIFFERENTIAL EQUATIONS FOR SOLVING THE CONSTANT K MATRIX.
**
      DERY(1)=-2.D0*(A(2,1)*Y(2)+A(4,1)*Y(4))-Q(1,1)+P(2,2)*Y(2)**2     SH003310
     *         +(P(2,4)+P(4,2))*Y(2)*Y(4)+P(4,4)*Y(4)**2                SH003320
```

```
      DERY(2)=-Y(1)-A(2,2)*Y(2)-A(4,2)*Y(4)-A(2,1)*Y(5)-A(4,1)*Y(7)      SH003330
     *        -Q(1,2)+P(2,2)*Y(2)*Y(5)+P(2,4)*Y(2)*Y(7)+P(4,2)*Y(4)*Y(5)SH003340
     *        +P(4,4)*Y(4)*Y(7)                                          SH003350
      DERY(3)=-A(2,3)*Y(2)-A(4,3)*Y(4)-A(2,1)*Y(6)-A(4,1)*Y(9)-Q(1,3)   SH003360
     *        +P(2,2)*Y(2)*Y(6)+P(2,4)*Y(2)*Y(9)+P(4,2)*Y(4)*Y(6)       SH003370
     *        +P(4,4)*Y(4)*Y(9)                                          SH003380
      DERY(4)=-A(2,4)*Y(2)-Y(3)-A(4,4)*Y(4)-A(2,1)*Y(7)-A(4,1)*Y(10)    SH003390
     *        -Q(1,4)+P(2,2)*Y(2)*Y(7)+P(2,4)*Y(2)*Y(10)+P(4,2)*Y(4)*   SH003400
     *        Y(7)+P(4,4)*Y(4)*Y(10)                                     SH003410
      DERY(5)=-2.D0*(Y(2)+A(2,2)*Y(5)+A(4,2)*Y(7))-Q(2,2)+P(2,2)*Y(5)**2SH003420
     *        +(P(2,4)+P(4,2))*Y(5)*Y(7)+P(4,4)*Y(7)**2                  SH003430
      DERY(6)=-Y(3)-A(2,3)*Y(5)-A(2,2)*Y(6)-A(4,3)*Y(7)-A(4,2)*Y(9)     SH003440
     *        -Q(2,3)+P(2,2)*Y(5)*Y(6)+P(2,4)*Y(5)*Y(9)+P(4,2)*Y(6)*Y(7)SH003450
     *        +P(4,4)*Y(7)*Y(9)                                          SH003460
      DERY(7)=-Y(4)-A(2,4)*Y(5)-Y(6)-(A(2,2)+A(4,4))*Y(7)-A(4,2)*Y(10)  SH003470
     *        -Q(2,4)+P(2,2)*Y(5)*Y(7)+P(2,4)*Y(5)*Y(10)+P(4,4)*Y(7)    SH003480
     *        *Y(10)+P(4,2)*Y(7)**2                                      SH003490
      DERY(8)=-2.D0*(A(2,3)*Y(6)+A(4,3)*Y(9))-Q(3,3)+P(2,2)*Y(6)**2     SH003500
     *        +(P(2,4)+P(4,2))*Y(6)*Y(9)+P(4,4)*Y(9)**2                  SH003510
      DERY(9)=-A(2,4)*Y(6)-A(2,3)*Y(7)-Y(8)-A(4,4)*Y(9)-A(4,3)*Y(10)    SH003520
     *        -Q(3,4)+P(2,2)*Y(6)*Y(7)+P(2,4)*Y(6)*Y(10)+P(4,2)*Y(7)*   SH003530
     *        Y(9)+P(4,4)*Y(9)*Y(10)                                     SH003540
      DERY(10)=-2.D0*(A(2,4)*Y(7)+Y(9)+A(4,4)*Y(10))-Q(4,4)+P(2,2)*Y(7)*SH003550
     *         *2+(P(2,4)+P(4,2))*Y(7)*Y(10)+P(4,4)*Y(10)**2             SH003560
      RETURN                                                            SH003570
***
***   STATE EQUATIONS FOR THE SEMISUBMERGED SHIP
***
    2 ZHATW=A1*DCOS(-WEFRQ*X)+B1*DSIN(-WEFRQ*X)                         SH003580
      MHATW=E1*DCOS(-WEFRQ*X)+D1*DSIN(-WEFRQ*X)                         SH003590
      IF(NKNTRL.EQ.0) GO TO 8                                           SH003595
      IF(KNTRL) 3,4,5                                                   SH003600
    3 U(1,1)=F(1,1)*Y(1)+F(1,2)*Y(2)+F(1,3)*Y(3)+F(1,4)*Y(4)+G(1,1)*    SH003610
     *        ZHATW+G(1,2)*MHATW                                         SH003620
      U(2,1)=F(2,1)*Y(1)+F(2,2)*Y(2)+F(2,3)*Y(3)+F(2,4)*Y(4)+G(2,1)*    SH003630
     *        ZHATW+G(2,2)*MHATW                                         SH003640
      GO TO 6                                                           SH003650
    4 U(1,1)=F(1,1)*Y(1)+F(1,2)*Y(2)+F(1,3)*Y(3)+F(1,4)*Y(4)            SH003660
      U(2,1)=F(2,1)*Y(1)+F(2,2)*Y(2)+F(2,3)*Y(3)+F(2,4)*Y(4)            SH003670
      GO TO 6                                                           SH003680
    5 V(1)=A2*DCOS(-WEFRQ*X)+B2*DSIN(-WEFRQ*X)                          SH003690
      V(2)=E2*DCOS(-WEFRQ*X)+D2*DSIN(-WEFRQ*X)                          SH003700
      U(1,1)=F(1,1)*Y(1)+F(1,2)*Y(2)+F(1,3)*Y(3)+F(1,4)*Y(4)+V(1)       SH003710
      U(2,1)=F(2,1)*Y(1)+F(2,2)*Y(2)+F(2,3)*Y(3)+F(2,4)*Y(4)+V(2)       SH003720
      GO TO 6                                                           SH003725
    8 U(1,1)=0.D0                                                       SH003726
      U(2,1)=0.D0                                                       SH003727
      GO TO 10                                                          SH003728
```

```
    6 IF(KSAT.LT.0) GO TO 10                                            SH003730
      DO 7 I=1,2                                                        SH003735
      IF(U(I,1).GT.0.523598775D0) U(I,1)=0.523598775D0                  SH003740
      IF(U(I,1).LT.-0.523598775D0) U(I,1)=-0.523598775D0                SH003750
    7 CONTINUE                                                          SH003760
   10 CALL DTRAN(Y,XT,4,1)                                              SH003770
      CALL DMPRD(Q,Y,QX,4,4,1)                                          SH003780
      CALL DMPRD(XT,QX,XTQX,1,4,1)                                      SH003790
      CALL DTRAN(U,JT,2,1)                                              SH003800
      CALL DMPRD(R,U,RU,2,2,1)                                          SH003810
      CALL DMPRD(UT,RU,UTRU,1,2,1)                                      SH003820
      DERY(1)=Y(2)                                                      SH003830
      DERY(2)=A(2,1)*Y(1)+A(2,2)*Y(2)+A(2,3)*Y(3)+A(2,4)*Y(4)           SH003840
     *        +B(2,1)*U(1,1)+B(2,2)*U(2,1)+0.4907D0*ZHATW-0.503D-1*MHATWSH003850
      DERY(3)=Y(4)                                                      SH003860
      DERY(4)=A(4,1)*Y(1)+A(4,2)*Y(2)+A(4,3)*Y(3)+A(4,4)*Y(4)           SH003870
     *        +B(4,1)*U(1,1)+B(4,2)*U(2,1)-0.502D-1*ZHATW+0.2865D0*MHATWSH003880
**
** COMPUTATIONS FOR THE PERFORMANCE MEASURE J
**
      DERY(5)=0.5D0*XTQX                                                SH003890
      DERY(6)=0.5D0*UTRU                                                SH003900
      RETURN                                                            SH003910
      END                                                               SH003920


      SUBROUTINE OUTP(X,Y,DERY,IHLF,NDIM,PRMT)                          SH003960
**
** THIS SUBROUTINE STORES, PRINTS, AND PLOTS THE DESIRED OUTPUTS FROM THE IBM
** SUBROUTINE DHPCG.
**
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH003970
      REAL*8 KMAT,JJ,JX,JT,MHATW                                        SH003980
      REAL*4 X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,TIME1,TIME2         SH003990
      REAL LABEL/'    '/                                                SH004000
      COMMON A(4,4),B(4,2),F(2,4),P(4,4),Q(4,4),RIBT(2,4),TEST,TEMP1(10)SH004010
     *,R(2,2),G(2,2),DEG,U(2,1),JU,JX,JT,MHATW,ZHATW,WEFRQ,A1,B1,D1,E1, SH004020
     *A2,B2,D2,E2,KNTRL,KSAT,NKNTRL,NK,II,JJ,LL                          SH004030
      DIMENSION Y(1),DERY(1),PRMT(1),KMAT(4,4),GAIN(4,4),AT(4,4),TITLE(1SH004040
     *2),X1(1000),X2(1000),X3(1000),X4(1000),X5(1000),X6(1000),TIME1(100SH004050
     *0),X7(1000),X8(1000),X9(1000),X10(1000),X11(1000),X12(1000),      SH004060
     *TIME2(1000)                                                       SH004070
      IF(LL.GT.2) GO TO 20                                              SH004080
**
** DETERMINE IF THE K MATRIX HAS REACHED ITS STEADY-STATE CONSTANT VALUE.
**
      DO 1 I=1,10                                                       SH004090
      CHECK=DABS(Y(I)-TEMP1(I))                                         SH004100
```

```
      IF(CHECK.GE.1.D-2) GO TO 9                                        SH004110
    1 CONTINUE                                                          SH004120
      II=II+1                                                           SH004130
      IF(II.GE.TEST) GO TO 2                                            SH004140
      RETURN                                                            SH004150
    9 TEMP1(I)=Y(I)                                                     SH004160
      II=0                                                              SH004170
      RETURN                                                            SH004180
**
**  STORE THE STEADY-STATE VALUES IN THE MATRIX GAIN(I,J)
**
    2 PRMT(5)=2.D0                                                      SH004190
      KJ=-1                                                             SH004200
      DO 4 I=1,4                                                        SH004210
      KJ=KJ+1                                                           SH004220
      IF(I.EQ.4) KJ=2                                                   SH004230
      DO 4 J=I,4                                                        SH004240
      IJ=I+J+KJ                                                         SH004250
      IF(I.EQ.1) IJ=J+KJ                                                SH004260
      KMAT(I,J)=-Y(IJ)                                                  SH004270
      KMAT(J,I)=KMAT(I,J)                                               SH004280
    4 CONTINUE                                                          SH004290
      DO 8 I=1,4                                                        SH004300
      DO 8 J=1,4                                                        SH004310
      GAIN(I,J)=-KMAT(I,J)                                              SH004320
    8 CONTINUE                                                          SH004330
**
**  COMPUTE THE CONSTANT STATE VECTOR FEEDBACK MATRIX F
**
      CALL DMPRD(RIBT,KMAT,F,2,4,4)                                     SH004340
      WRITE(6,60)                                                       SH004350
      DO 5 I=1,2                                                        SH004360
      WRITE(6,55) (I,J,F(I,J),J=1,4)                                    SH004370
    5 CONTINUE                                                          SH004380
      WRITE(6,60)                                                       SH004390
      DO 7 I=1,4                                                        SH004400
      WRITE(6,65)(I,J,GAIN(I,J),J=1,4)                                  SH004410
    7 CONTINUE                                                          SH004420
      RETURN                                                            SH004430
   20 IF(LL.GT.5) GO TO 30                                              SH004440
      IF(JJ.LT.1) GO TO 21                                              SH004450
      JJ=JJ+1                                                           SH004460
      IF(JJ.GE.14) JJ=0                                                 SH004470
      RETURN                                                            SH004480
   21 II=II+1                                                           SH004490
      JJ=JJ+1                                                           SH004500
      IF(NK.LT.2) GO TO 23                                              SH004510
**
```

```
**  RUN #1 TIME HISTORIES--
**                          X1=>HEAVE
**                          X2=>HEAVE VELOCITY
**                          X3=>PITCH
**                          X4=>PITCH VELOCITY
**                          X5=>ELEVATOR CONTROL SURFACE
**                          X6=>CANARD CONTROL SURFACE
**
      X1(II)=Y(1)                                                       SH004520
      X2(II)=Y(2)                                                       SH004530
      X3(II)=Y(3)*DEG                                                   SH004540
      X4(II)=Y(4)*DEG                                                   SH004550
      X5(II)=U(1,1)*DEG                                                 SH004560
      X6(II)=U(2,1)*DEG                                                 SH004570
      TIME1(II)=X                                                       SH004580
      GO TO 25                                                          SH004590
**
**  RUN #2 TIME HISTORIES--
**                          X7 =>HEAVE
**                          X8 =>HEAVE VELOCITY
**                          X9 =>PITCH
**                          X10=>PITCH VELOCITY
**                          X11=>ELEVATOR CONTROL SURFACE
**                          X12=>CANARD CONTROL SURFACE
**
   23 X7(II)=Y(1)                                                       SH004600
      X8(II)=Y(2)                                                       SH004610
      X9(II)=Y(3)*DEG                                                   SH004620
      X10(II)=Y(4)*DEG                                                  SH004630
      X11(II)=U(1,1)*DEG                                                SH004640
      X12(II)=U(2,1)*DEG                                                SH004650
      TIME2(II)=X                                                       SH004660
**
**  COMPUTE THE PERFORMANCE MEASURE JT (J TOTAL)
**
   25 JX=JX+Y(5)                                                        SH004670
      JU=JU+Y(6)                                                        SH004680
      JT=JX+JU                                                          SH004690
      IF(II.EQ.1000) PRMT(5)=1.D0                                       SH004700
      RETURN                                                            SH004710
**
**  THIS PART OF THE PROGRAM MAY BE USED FOR PRINTING OR PLOTTING THE OUTPUT.
**  NPS SUBROUTINE DRAW WAS USED DURING THE INVESTIGATION.
**
   30 READ(5,90) TITLE                                                  SH004840
      CALL DRAW(II,TIME1,X1,1,0,LABEL,TITLE,10.,.2,3,RT,2,0,8,4,0,Z)
      CALL DRAW(II,TIME2,X7,3,0,LABEL,TITLE,10.,.2,3,RT,2,0,8,4,0,Z)
      READ(5,90) TITLE                                                  SH004880
```

```
      CALL DRAW(II,TIME1,X3,1,0,LABEL,TITLE,10.,10.,1,RT,2,0,8,4,0,Z)
      CALL DRAW(II,TIME2,X9,3,0,LABEL,TITLE,10.,10.,1,RT,2,0,8,4,0,Z)
      WRITE(6,86) II                                                    SH005090
   55 FORMAT(4(4X,'F(',I1,',',I1,') =',D16.8),/,/)                      SH005100
   60 FORMAT(/,/,/,/)                                                   SH005110
   65 FORMAT(4(4X,'K(',I1,',',I1,') =',D16.8),/,/)                      SH005120
   75 FORMAT('1')                                                       SH005130
   84 FORMAT('1',T9,'TIME1',T28,'HEAVE',T40,'HEAVE VELOCITY',T64,'PITCH'SH005140
     *        ,T76,'PITCH VELOCITY',T97,'ELEVATOR',T117,'CANARD',/,/)   SH005150
   85 FORMAT(7(F16.8,2X))                                               SH005160
   86 FORMAT(20X,'II =',I4)                                             SH005170
   90 FORMAT(6A8)                                                       SH005210
   91 FORMAT(A4)                                                        SH005220
      RETURN                                                            SH005240
      END                                                               SH005250

      SUBROUTINE DRINV(A,B)                                             SH005290
**
*** THIS SUBROUTINE COMPUTES THE DOUBLE PRECISION INVERSE OF A (2 X 2) MATRIX A
*** AND RETURNS THE RESULT IN THE (2 X 2) MATRIX B.
**
      IMPLICIT REAL*8 (A-H,O-Z)                                         SH005300
      DIMENSION A(2,2),B(2,2)                                           SH005310
      DEL=A(1,1)*A(2,2)-A(1,2)*A(2,1)                                   SH005320
      B(1,1)=A(2,2)/DEL                                                 SH005330
      B(1,2)=-A(1,2)/DEL                                                SH005340
      B(2,1)=-A(2,1)/DEL                                                SH005350
      B(2,2)=A(1,1)/DEL                                                 SH005360
      RETURN                                                            SH005370
      END                                                               SH005380

      SUBROUTINE EST(A,B,C,D,I)                                         SH005420
**
*** THIS SUBROUTINE RETURNS THE COEFFICIENTS OF THE COMMAND SIGNAL V(T) WHEN
*** UHAT IS 1.0
**
      IMPLICIT REAL*8 (A-H,O,Z)                                         SH005430
      IF(I) 1,2,3                                                       SH005440
**
*** THE R MATRIX IS ONE TENTH OF THE IDENTITY MATRIX
**
    1 A=0.19135600D0*DSIN(3.204156965D0)                                SH005450
      B=-0.19135600D0*DCOS(3.204156965D0)                               SH005460
      C=0.17333513D0*DSIN(0.477839366D0)                                SH005470
      D=-0.17333513D0*DCOS(0.477839366D0)                               SH005480
      RETURN                                                            SH005490
```

```
**
***  THE R MATRIX IS ONE HALF OF THE IDENTITY MATRIX
**
    2 A=0.12337738D0*DSIN(3.405767837D0)                                SH005500
      B=-0.12337738D0*DCOS(3.405767837D0)                               SH005510
      C=0.10170269D0*DSIN(0.342135621D0)                                SH005520
      D=-0.10170269D0*DCOS(0.342135621D0)                               SH005530
      RETURN                                                            SH005540
**
***  THE R MATRIX IS THE IDENTITY MATRIX
**
    3 A=0.9399569D-1*DSIN(3.432566603D0)                                SH005550
      B=-0.9399569D-1*DCOS(3.432566603D0)                               SH005560
      C=0.7081240D-1*DSIN(0.218452002D0)                                SH005570
      D=-0.7081240D-1*DCOS(0.218452002D0)                               SH005580
      RETURN                                                            SH005590
      END                                                               SH005600
```

LIST OF REFERENCES

1. Naval Undersea Research and Development Center Technical Note TN-414, Naval Feasibility Study of the NUC Semi-bmerged Ship Concept, Part 1, by T. G. Lang, September 1970.

2. Abkowitz, M. A., Stability and Motion Control of Ocean Vehicles, M.I.T. Press, 1969.

3. Naval Undersea Research and Development Center Technical Note TN-582, Dynamics and Control of a Twin-Hulled Semisubmerged Ship, by D. T. Higdon, 1 August 1971.

4. Ward, J. R. and Strum, R. D., The Signal Flow Graph in Linear Systems Analysis, Prentice-Hall, 1968.

5. Kirk, D. E., Optimal Control Theory, An Introduction, Prentice-Hall, 1970.

6. Kalman, R. E., "Contributions to the Theory of Optimal Control," Bol. Soc. Mat. Mex., 1960.

7. Athans, M. and Falb, P.L., Optimal Control, p. 773, 801, McGraw-Hill, 1966.

8. Hsu, J.C. and Meyer, A.U., Modern Control Principles and and Applications, p. 342, McGraw-Hill, 1968.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Professor Donald E. Kirk, Code 52 Ki Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	4
4.	Professor Harold A. Titus, Code 52 Ts Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	6
5.	Dr. Donald T. Higdon Advanced Concepts Group Naval Undersea Research and Development Center San Diego, California 92106	1
6.	LT Gilbert F. Monell, Jr., USN 3508 Schoolhouse Lane Harrisburg, Pennsylvania 17109	1

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

AUTOMATIC CONTROL SYSTEMS
FOR
LONGITUDINAL MOTION OF SEMISUBMERGED SHIPS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Electrical Engineer Thesis; Sepbember 1972

5. AUTHOR(S) *(First name, middle initial, last name)*

Gilbert Finley Monell, Jr.
Lieutenant, United States Navy

6. REPORT DATE

September 1972

7a. TOTAL NO. OF PAGES

113

7b. NO. OF REFS

8

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School
Monterey, California 93940

13. ABSTRACT

Optimal control theory is used to develop three automatic control systems for the longitudinal motion of a semisubmerged ship. A linearized mathematical model of the ship motion is used and the control problem is treated as a linear regulator. Simulations of the ship's longitudinal motions, utilizing the three control systems, are compared for various sea conditions. It is concluded that if the wave forces and moments are known or are estimated as functions of time, a suboptimal controller is the best controller. However, if a frequency domain approach is used to estimate the wave forces and moments, the complexity of a suboptimal controller approaches that of an optimal controller and the difference between the two is negligible. Preliminary work on three techniques for estimating the forces and moments is presented.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Unclassified

Security Classification

A-31408

Unclassified

Security Classification

14 KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Semisubmerged Ship						
Optimal Control						
Suboptimal Control						
Linear Regulator						
Estimator						
Observer						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Unclassified

Security Classification

A-31408

5 NOV 74
25 MAY 76
12 MAR 79
22793
23200
25275

Thesis
M673
c.1
137134
Monell
Automatic control systems for longitudinal motion of semisubmerged ships.
5 NOV 74
25 MAY 76
12 MAR 79
22793
23200
25275

Thesis
M673
c.1
137134
Monell
Automatic control systems for longitudinal motion of semisubmerged ships.

thesM673
Automatic control systems for longitudin
3 2768 001 01227 1
DUDLEY KNOX LIBRARY